Effects of student-reported gameplay strategy related to growth in multiplicative reasoning.

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Abstract

Multiplicative reasoning is essential for developing meaningful understandings of fractions, rational number, and algebraic relationships. Several mobile applications are available for parents and teachers to download and supplement elementary students' learning experiences with multiplication and division, and some studies support the use of various apps for improving students' mathematical skill. As valuable as quasi-experimental studies of app effectiveness are, there is often a need for further detail regarding not whether an app can improve students' mathematical understanding, but in what conditions such change occurs. The present study examined fifth grade students' reported strategy use to examine the effect on growth in multiplicative reasoning. Findings suggest that students who focus on more advanced unit coordination (units of units of units) demonstrated positive growth, whereas students who coordinated less complex units (units of units) did not demonstrate growth.

1. Introduction

U.S. tablet ownership has increased from a paucity 3% in 2010 to 45% in 2015 [22], with 58% of teens reporting access to tablets [23]. Likewise, schools are increasingly adopting tablets, albeit with a range of available devices per student and/or classroom [24]. There are thousands of apps available that focus on education, but the majority of such apps focus on the presentation of information rather than engaging the student in interacting with and/or constructing their own knowledge [18]. This trend is prevalent with mathematics education, with the vast majority of such mobile applications focusing on the memorization of mathematics facts [15, 31]. Research on the effectiveness of mathematics apps has found that students who use mathematics-focused apps tend to have higher achievement scores than those who do not [3, 5, 11, 13, 25, 26]. The vast majority of such research examines apps that engage students in constructing and/or interacting with the mathematics. However, some of these studies include apps that focus on memorization. Advocates of reform in mathematics education argue that incorporation of technology should include a connection to conceptual understanding as opposed to rotely learned procedures [4, 19]. A limitation of much of the current research is that it focuses on app usage as a treatment that does or does not facilitate learning, but does not fully account for students' actions and interactions with such apps that may or may not facilitate such learning.

Various mathematics apps include features that are meant to facilitate engagement in specific strategies or concepts [1, 2, 3, 16, 17]. However, such features may not be adopted by all students engaging with the app [1]. Although the vast majority of research on apps in education, within and beyond the context of mathematics, focuses on the use of particular apps or specific features within such apps, there has been relatively little focus on the different strategies used by students when interacting with such apps. Specifically, although particular features may be included to scaffold student actions within an app, there is an element of choice in the strategy a student may use in engaging with the app. Such choices may have important implications regarding the effectiveness of using the app on learning outcomes, and provide important implications for teacher use in the classroom and/or revision of the app interface itself.

The purpose of the present study is to investigate the strategies of fifth grade students in playing an app designed to facilitate multiplicative reasoning: *CandyDepot* [20], and when such strategies are more (or less) beneficial for students' demonstrated multiplicative reasoning. Although the majority of U.S. States teach basic multiplication and division in third grade, concepts relying on multiplicative reasoning are taught across the elementary grades [Y]. For example, fraction operations focused on in fourth and fifth grade rely heavily on students' understanding of certain forms of multiplicative reasoning [5, 8, 10]. Thus, multiplicative reasoning is an important concept of focus that transcends various other mathematical concepts and topics. The *CandyDepot* app provides a context in which students can transition from less to more sophisticated forms of multiplicative reasoning. Such a transition is likely to occur when students use particular types of strategies within the mobile app environment. The present study focused on whether shifts in multiplicative reasoning were related to students' reported strategy use in *CandyDepot*.

2. Mobile Gaming and Mathematics learning

An increasing number of studies have found that when students engage with a particular mathematics app, they have higher mathematics achievement scores as compared to students who did not engage with the particular app. The specific focus of such apps include examination of: preschool children's playing of *Zorbit's Math Adventure for Preschool* [13]; third grade students' playing of *Wuzzit Trouble* [25]; fifth grade students' playing of *Motion Math* [26]; and middle grades students' use of *CandyFactory* [3]. Individually, the findings of such research indicate that these specific apps are more beneficial to students' mathematics achievement than not playing the apps. Although a necessary step in evaluating whether such apps meet a goal of benefitting students' learning, few of these studies dig deeper than whether there is growth in an arbitrary score. Exceptions include supplemental analysis of preschool students' playing the *Zorbit's Math Adventures* app [13]. While students who played the app saw significant growth, playing the app appeared to have an additive, but not interacting, effect with other forms of productive play, such as puzzle play, in terms of children's learning. Another example suggests that while playing *CandyDepot* improved middle grades students' understanding of fractions, the app was particularly effective for "inclusion students" [3]. Another study focused on 15 early elementary students playing *Addimal Adventure*, and found that playing the app encouraged students to use specific strategies for solving addition/subtraction tasks in mathematics [2]. Specifically, although students' mathematics achievement improved over the course of using the app, the students' specific strategy use in solving addition and subtraction problems concurrently changed from less to more sophisticated strategies. Although focused on particular mathematics mobile apps, such research goes beyond whether or not a particular app can improve mathematics achievement towards how and in what ways such apps benefit children's mathematics learning.

Much of research on students' use of mathematics apps focuses on how mobile apps affect students. Relatively little study has focused on how students interact with specific apps. An exception to this trend is a recent study of various virtual manipulative mathematics apps and provide a useful exception to this trend [17]. The research team interviewed 100 children across grades K-2 as the children played various apps given specific goals for the grade level. Certain apps were specific to a particular grade level, while others were used across grade levels. Findings revealed that for some apps, features within the apps were used different by students at different grade levels. Specifically, "affordances of technology apps can support learning performance and efficiency for some children (when accessed), can hinder, slow down or frustrate some children (when accessed), or can be ignored by other children (meaning that they did not access the affordance at all)" [17]. A main implication from this finding is that when children access the specific features designed to promote growth, such children saw more growth in achievement than others. However, observations of students' use of the apps did not attend to students' perception of such affordances [17]. This latter facet is one in which the present study focuses. Specifically, the present study sought to investigate whether students decided to use certain features within the *CandyDepot* app that were designed to promote growth in multiplicative reasoning.

3. Children's multiplicative reasoning

The present study considers multiplicative reasoning from the perspective of scheme theory, with particular attention to descriptions of the multiplicative concepts [7, 8]. A scheme is a set of mental actions with a goa-oriented purpose [30]. Mathematics education research has typically focused on actions such as unitizing, iterating, partitioning, and disembedding. Within multiplicative contexts: *unitizing* involves conveying a set of discrete countable objects as a countable unit in its own right; *iterating* involves the repetition of a unit (typically via counting or skip-counting); *partitioning* involves the separation of a unit into smaller sub-units, and *disembedding* involves taking a composite unit (unit of units) from another unit while considering the new unit relationally to the original unit [7, 27]. The coordination of specific actions leads to the construction of schemes, some of which are multiplicative in nature [27, 28]. Multiplicative concepts include the coordination of constructed multiplicative schemes [7]. Each multiplicative concept involves the use of schemes that anticipate and engage with abstraction of levels of units within an activity.

There are three multiplicative concepts [8]. The first multiplicative concept (MC1) involves the coordination of two levels of units in activity. Students at MC1 can consider a given unit and coordinate between two units. For example, in solving 5 x 3, a student at MC1 may count by 1s to 15 by keeping track of how many counts of 5 they use ("1,2,3…4,5,6…7,8,9.. etc."). Students at the second multiplicative concept (MC2) can coordinate three levels of units in activity [10]. For example, if asked how many more 3s there are in 27 than 15, a student at MC2 can disembed 3 from 15 and operate on the remaining 12 (27 – 15). This interaction takes-as-given two levels of units by operation on 3s as an object. At the third multiplicative concept (MC3), students coordinate three levels of units, as with MC2, but do so in a way that assumes all three levels a prior. This allows for more sophisticated strategies. For example, a student at MC3 can meaningfully relate and use the factorization of 12 and 60 by relating 12 as 3×4 and 60 as $(3 \times 4) \times 5$. A student at MC2 would have difficulty relating the two factors in 12 with the three factors in 60. Thus, each multiplicative concept involves the coordination of units at different levels and with different degrees of abstraction.

Research studying the multiplicative concepts of K-8 students has found that students rely on their multiplicative reasoning to operate with fractions [7, 8, 21]. Students at MC1 might be able to represent the fraction three-fourths by partitioning a shape into four parts and identifying three parts within the whole as the requested fraction, but the student does not consider these three parts as a singular entity (but as a count of three parts). Students at MC2 might be able to represent the same fraction, but because they are able to disembed may recognize the three parts as an entity representing a singular number [8]. Similar to research on fractions, research on algebraic reasoning has found that higher multiplicative concepts facilitate more sophisticated activity with equations [9]. However, as many as half of third grade students are below MC1 [14], and this trend is estimated to continue even through fifth grade [29].Thus, there is a critical need for resources for students to improve their ability to coordinate units multiplicatively.

4. *CandyDepot*: An application for developing multiplicative reasoning

CandyDepot is a mobile mathematics application designed by the Learning Transformation Research Group at Virginia Tech [20]. Specifically, the app is designed to support students' developed unit coordination for multiplicative concepts 2 and 3. The game-based app is situated in the context of a packing depot in which the player is tasked with fulfilling customer orders by using bars, bundles, and boxes. Bundles include collections of bars and boxes include collections of bundles. Prior to play, the player enters how many bars per bundle and bundles per box they will play the level. The player is then presented with different customer orders and is tasked with fulfilling the orders in an allotted amount of time, and with the most efficient package possible. For example, if the player is playing a level in which a box is worth two bundles and a bundle is worth three bars, they may be asked to fulfil an order for five bundles. The player can choose to drag five individual bundles into the delivery truck, or they can choose to drag two boxes and one bundle into the truck. While the former is a simple enough means of fulfilling the order, the second strategy is more efficient and provides the player with a higher score. Customer orders can come in the form of whole numbers or fractions of bars, bundles, or boxes.

Prior study of the app has found that students playing the app tend to improve their multiplicative unit coordination [20]. However, guidelines for the app recommend that it be incorporated into classroom instruction through students' recording of strategies and discussion of those strategies. To this end, there is a teacher handbook for grades 4 to 7 available online via a link in the app store for the game. Therefore, the app is intended for integrated classroom use. The present study investigated fifth grade students' outside of this recommended context. Specifically, the present study sought to investigate students' choice of strategy in playing the game. Doing so in a context in which a teacher may scaffold such strategies would detract from the primary goal of this study, and such a context was therefore excluded. Rather than examining the effectiveness of the app itself (which has already been demonstrated by prior study), the focus in this study was to investigate whether students' varying strategies in regards to the preferred use of boxes, bundles, or bars in fulfilling orders would affect their developed multiplicative reasoning. It is hypothesized that strategies focused on box use over bundles or bars would lead to more growth in unit coordination, given the design of the game and nature of multiplicative reasoning itself. Therefore, we asked the following research question:

> *Is there a relationship between students' reported strategy use and the observed change in their multiplicative reasoning?*

5. Methods

5.1 Data and Measures

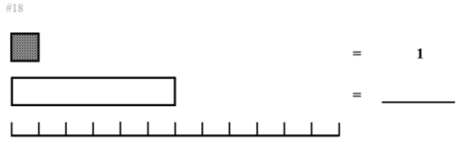
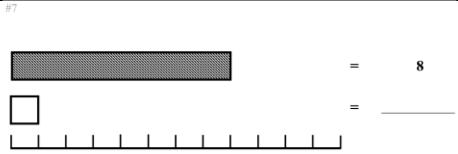
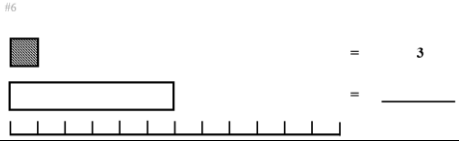
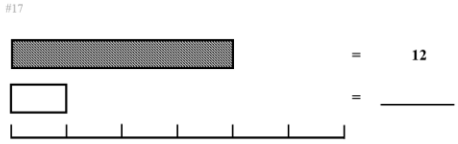
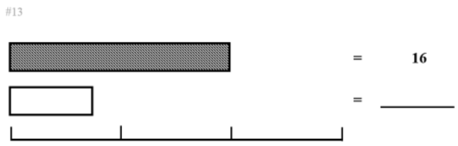
Data were collected from 23 fifth-grade students from a single class in a Midwestern U.S. state. The teacher and students participated in an immersive professional development experience in which they visited a technology-rich classroom for half a day over a six week period. The classroom included numerous digital cameras to record video and audio at multiple points in the room, as well as a one-way mirror for researchers to observe the classroom in a neighbouring room. A major goal of the professional development experience was to provide teachers with numerous experiences and on-site support for integrating technology in novel ways, so that they could do so effectively when returning to their own school and classroom. One of the technologies of interest by the teacher was use of tablets in mathematics. The present study reports on data collected in this regard. Given the size of the sample, and circumstances of data collection (i.e., within a larger professional development program), this study does not report on a representative sample and interpretations of findings should be considered in this respect.

The author provided students (and their teacher) with an overview of how to play the *CandyDepot* during the first week. Specifically, the author displayed his screen of *CandyDepot,* removing the timed feature of play, and guided the class in a discussion of how to fulfil orders in Level 1 when the rule was that there are 2 bars per bundle and 2 bundles per box. For example, provided an order for 13/2 bundles, the author asked students to discuss how they might fill that order. Student responses included a set number of bars while other students recommended a combination of bundles and bars. The author then reminded students of the context of the game (i.e., that they worked at a factory) and that their 'boss' would want them to use the fewest number of containers possible. This facilitated a brief discussion about the combination of bars, bundles, and boxes needed to have the fewest containers possible (i.e., 3 boxes and 1 bar). After several orders had been discussed and filled as a whole class, students were tasked with playing the app on their own with the same rule (i.e., 2 bars per bundle; 2 bundles per box) for a 15 minute period. If students finished the level early, they were invited to create their own rule to try in the game until time for playing *CandyDepot* expired. The author, classroom teacher, and a support staff member walked the classroom to answer any student questions regarding gameplay, and to generally facilitate and troubleshoot students' first play of the app.

After this initial play, students were tasked with playing the app for three days a week, 15 minutes per day, for six weeks. For each day, students were provided with an initial rule (e.g., for day 3 on week 3, students were given the initial rule of 4 bars per bundle and 5 bundles per box). Thus, over the course of six weeks, students accumulated 4.5 hours of game play. By focusing students' introduction to *CandyDepot,* and providing an initial rule for each engagement with the app, the goal was to provide students with the necessary understanding of the game context itself so that a study of their self-reported strategy use within this context would have less to do with their game knowledge and more to do with their mathematical understanding.

Participating students completed a pre- and post-test assessing their multiplicative reasoning prior to playing the app and following the last scheduled playing of the app. The multiplicative reasoning assessment includes 19 items and was designed to assess participants' demonstrated multiplicative concepts [12, 14]. The initial validation of the assessment observed sufficient internal reliability ($\alpha = .79$) and also included a comparative analysis of students' strategies with their responses. Specifically, students with a correct response on items also demonstrated strategies that aligned with the targeted multiplicative scheme [14]. However, the original assessment examined multiplicative reasoning only up to the second multiplicative concept (MC2). The assessment used in the present study is a revised version of the original assessment, including additional items to assess MC3. The assessment was validated using Rasch modelling and has sufficient item reliability (.93) and person reliability (.86). Additionally items were found to have acceptable mean square infit statistics, indicating that items appear to measure the construct reliably [12]. Table 1 includes example items from the pre-test of the assessment. Items from both the pre- and post-test followed the same format. For example, students who demonstrate multiplicative reasoning at MC1 might be able to solve some tasks targeting MC2, but because they rely on less sophisticated strategies, they do so with significantly lower success [12, 14]. More specific details on the nature of the assessment is provided in the validation studies of the two versions [12, 14]. The present form of the assessment did demonstrate internal reliability consistent with these validations studies for both the pre-test ($\alpha = .87$) and post-test ($\alpha = .84$).

Table 1. *Example items from the multiplicative reasoning assessments*.

Level	# Items	Description of Students' Item Interaction	Example Items
Pre-Mult.	3	Must count-by-1s, and must coordinate one level of units in activity.	
			 
			 
			 

Using a one-tailed paired samples t-test, the difference between pre- (*M* = .34, *SD* = .22) and post-test (*M* = .42, *SD* = .20) was found to be statistically significant at the .05 level (t = 2.03, p = .028). With few exceptions, students' number of correct responses corresponded to the item hierarchy illustrated in Table 1 (recall that prior research found strong correlations with strategy use and response per item type). When classifying participants' responses in terms of the multiplicative concepts, at pre-test 40.7% of participants demonstrated pre-multiplicative reasoning and 14.8% demonstrated MC1. This corresponds with estimates of approximately half of fifth grade students demonstrating at most an iterative multiplicative scheme [29]. However, at post-test, these percentages were reduced to 23.1% at pre-multiplicative and 11.5% at MC1. Given prior research on usage of the app [20], growth in multiplicative reasoning was anticipated. Rather, the present study focuses on explaining such growth and these statistics. Therefore, while these statistics are encouraging, they are only meant to be descriptive in nature.

In addition to the pre- and post-test, participants were asked to complete a set of survey questions, of which three post-survey questions were included in the present study. Participants were asked to rate how frequently they performed different actions when playing CandyDepot (0 – *Never or*

Hardly Ever; 1 – *Less than Half of the Orders*; 2 – *More than Half of the Orders*; 3 – *Always or Almost Always*). The items were nearly identical, including the statements:

- I used bars to fill customer orders (*M* = 1.08, *SD* = 1.04).
- I used bundles to fill customer orders (*M* = 1.60, *SD* = .91).
- I used boxes to fill customer orders (*M* = 2.56, *SD* = .58).

As evident from the descriptive statistics, there was a more frequent reported use of using boxes than bundles, and bundles than bars. Important to note here is that the present study relied on students' self-reported items. Such data may align with actual behaviour [6], but should not be interpreted as identical to observed behaviour. Although data from video recordings in the classroom was available for analysis to examine students' observable behaviour, the present analysis purposefully used student self-reported data. Specifically, regardless of whether a student used boxes frequently or not in their actual gameplay, their perception of such use describes the student's own organization of multiplicative structures in the game at the same point of assessment. By contrast, observations of students' actions in playing *CandyDepot* must be interpreted from the researcher's perception of student engagement. While both perspectives hold certain value, the present study focused on the student perspective as a preliminary indicator for how such strategies may relate to cognitive growth.

5.2 Analysis and Results

The purpose of the present study is to investigate when and whether certain student-reported strategies relate to improved multiplicative reasoning in playing *CandyDepot*. Given the sample size and stated goals of the study, one-tailed partial Spearman Rho correlation coefficients were calculated for this purpose. The correlation coefficients were controlled for pre-test scores, and were calculated between students' post-test multiplicative reasoning score and their reported use of bars to fill customer orders ($\rho = -.22$, $p = .153$); reported use of bundles to fill customer orders ($\rho = -.30$, $p = .075$); and reported use of boxes to fill customer orders ($\rho = .33$, $p = .056$). Bundle and box usage were found to be statistically significant at the .10 level, while bar usage was not found to be statistically significant from chance. Therefore, when accounting for pre-test scores, participants who reported a higher usage of bundles to fill customer orders tended to have lower post-test scores and those reporting lower use of bundles had higher post-test scores. By contrast, participants who reported a higher usage of boxes to fill customer orders tended to have higher post-test scores, and vice versa. Thus, in answer to the research question of whether there is a relationship between participants' reported strategy use and their observed change in multiplicative reasoning, the short answer is yes, but with specific relationships associated with particular changes.

In order to better understand the partial correlation statistics, a supplementary analysis of the post-test scores was conducted for participants who reported using boxes "always or almost always" to fulfil customer orders in *CandyDepot*. Spearman Rho correlations were calculated for these participants for relationships between post-test scores with reported bundle usage ($\rho = -.48$, $p = .070$) and reported bar usage ($\rho = -.33$, $p = .233$). Findings indicate that those participants who report using boxes "always or almost always" tend to have higher post-test scores when they also report using bundles less frequently, and vice versa. Thus, it appears that while a preference for using boxes as a strategy for fulfilling customer orders is positively correlated with higher post-test scores, when accounting for pre-test scores of multiplicative reasoning, a preference for using bundles is negatively correlated. Furthermore, for those students who report higher use of boxes as a strategy, it appears that a preference for using bundles may have a negative effect on overall post-test scores.

5.3 Limitations and Caveats

The results of the present study indicate that students who report using boxes more prevalently also tended to see the most growth in their multiplicative reasoning. However, these results are based on a small sample (n = 23) and should be interpreted as preliminary. Further, while the self-report data provided by students is useful, it is only one means of examining strategy use. Other forms of examining strategy use (i.e., observations of student play, data collected from the app itself) were not included in the present analysis. Thus, the results of the present study should be interpreted in context with these limitations.

6. Conclusions

CandyDepot is an effective mobile mathematics application for facilitating students' improved multiplicative unit coordination [20]. The present study sought to preliminarily investigate whether students' preferred strategies for playing the app affected their growth in multiplicative reasoning. Findings suggest a positive statistical correlation between the frequency students reported using boxes to fulfil customer orders and higher post-test scores (when accounting for pre-test scores). However, preference for using bundles was found to have a negative relationship with post-test scores. Furthermore, students who reported using boxes more frequently saw more growth when they reported using bundles less frequently, and vice versa. By design, *CandyDepot* is meant to encourage the use of more efficient strategies to scaffold MC2 and MC3 for multiplicative reasoning. Such efficient strategies include a preferred use of boxes over bundles. Thus, the findings of the present study support this feature of the app's design. However, findings indicate that many students reported using bundles more than boxes, or using both at similar rates. Since the app is advocated for use in classrooms with teacher scaffolding, a significant implication from this study is that teachers advocate the use of boxes as frequently as is feasible to fulfil customer orders. A general implication from the present study for other mobile mathematics apps is that the students' preference for strategy in engaging with the app matters. As found in prior study [17], the scaffolds for supporting mathematical learning included in many apps tend to work when used by students. However, not all students will use such scaffolds on their own. Thus, a design implication for mobile mathematics apps is to give serious consideration to supporting students' engagement with the scaffolds designed to facilitate learning.

Further study is necessary to better understand how scaffolds embedded within mobile applications can be supported both within specific apps and as a general design guideline across apps. There are several methodological challenges to such investigations. First, the manner in which strategy use is assessed may stem from student reports, as in the present study, or through observations of student actions interpreted by researchers [16, 17]. Both methods have merit but also limitations and reports of either should be interpreted accordingly. Second, the apps used for comparison should be considered in depth. For example, if seeking to examine student engagement with scaffolds for multiplicative reasoning, it is not sufficient to choose apps simply because they address multiplication and/or division. Rather, the design of such apps should follow similar design principles and philosophies, or the contexts which scaffolds are embedded may be too dissimilar for meaningful interpretation of students' actions (but may be appropriate for comparisons of the apps themselves). The present study, although preliminary and limited in scope, provides an example of how such studies may be framed, expanded upon, and replicated.

7. Author Notes

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8. References

[1] Bullock, E. P., Moyer-Packenham, P. S., Shumway, J. F., MacDonald, B., & Watts, C. (2015). Effective teaching with technology: Managing affordances in iPad apps to promote young children's mathematics learning. In *Society for Information Technology & Teacher Education International Conference* (vol. 2015, no. 1, pp. 2648-2655). Las Vegas, NV.

[2] Carpenter, K., Pagar, D., & Labrecque, R. (2013). *Teachley: Addmial adventure: Bridging research and technology to help children foster strategy development, conceptual understanding, and number fact fluency* (White Paper). Retrieved at: http://www.teachley.com/assets/docs/White-Paper-Addimal-Adventure.pdf

[3] Chang, M., Evans, M. A., Kim, S., Norton, A., & Samur, Y. (2015). Differential effects of learning games on mathematics proficiency. *Educational Media International, 52*(1), 47-57.

[4] Common Core State Standards Initiative [CCSSI]. (2010). Common Core State Standards for Mathematics. Retrieved at: http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf

[5] Franklin, T., & Peng, L. (2008). Mobile math: Math educators and students engage in mobile learning. *Journal of Computing in Higher Education, 20*, 69-80.

[6] Fredricks, J. A., & McColskey, W. (2012). The measurement of student engagement: A comparative analysis of various methods and student self-report instruments. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.), *Handbook of research on student engagement* (pp. 763-782). New York: Springer.

[7] Hackenberg, A. J. (2010). Students' reasoning with reversible multiplicative relationships. *Cognition and Instruction, 28*(4), 383–432.

[8] Hackenberg, A. J. (2013). The fractional knowledge and algebraic reasoning of students with the first multiplicative concept. *Journal of Mathematical Behavior, 32*(3), 538–563.

[9] Hackenberg, A. J., & Lee, M. Y. (2015). How does students' fractional knowledge influence equation writing? *Journal for Research in Mathematics Education, 46*(2), 199–243.

[10] Hackenberg, A. J., & Tillema, E. S. (2009). Students' whole number multiplicative concepts: A critical constructive resource for fraction composition schemes. *Journal of Mathematical Behavior, 28*(1), 1-18).

[11] Kiger, D., Herro, D., & Prunty, D. (2012). Examining the influence of a mobile learning intervention on third grade math achievement. *Journal of Research on Technology in Education, 45*(1), 61-82.

[12] Kosko, K. W. (in review). A multiplicative reasoning assessment for fourth and fifth grade students.

[13] Kosko, K. W., & Ferdig, R. E. (2016). Effects of a tablet-based mathematics application for pre-school children. *Journal of Computers in Mathematics and Science Teaching, 35*(1), 61-79.

[14] Kosko, K. W., & Singh, R. (in press). Elementary children's multiplicative reasoning: Initial validation of a written assessment. *The Mathematics Educator*.

[15] Larkin, K. (2013). Mathematics education: Is there an app for that? In V. Steinle, L. Ball, & C. Bardini (Eds.), *Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia* (pp. 426-433). Melbourne, Australia: MERGA.

[16] Moyer-Packenham, P. S., Bullock, E. K., Shumway, J. F., Tucker, S. I., Watts, C. M., Westenskow, A., Anderson-Pence, K. L., Maahs-Fladung, C., Boyer-Thurgood, J., Gulkilik, H., Jordan, K. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal, 28*, 79-105.

[17] Moyer-Packenham, P.S., Shumway, J.F., Bullock, E., Tucker, S.I., Anderson-Pence, K.L., Westenskow, A., Boyer-Thurgood, J., Maahs-Fladung, C., Symanzik, J., Mahamane, S., MacDonald, B. & Jordan, K. (2015). Young Children's Learning Performance and Efficiency when Using Virtual Manipulative Mathematics iPad Apps. *Journal of Computers in Mathematics and Science Teaching, 34*(1), 41-69.

[18] Murray, O., & Olcese, N. (2011). Teaching and learning with iPads, Ready or not? *Techtrends: Linking Research & Practice to Improve Learning, 55*(6), 42-48. doi:10.1007/s11528-011-0540-6

[19] National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.

[20] Norton, A., Boyce, S., & Hatch, J. (2015). Coordinating units at the CandyDepot. *Mathematics Teaching in the Middle School, 21*(5), 280-287.

[21] Norton, A., Boyce, S., Ulrich, C., & Phillips, N. (2015). Students' units coordination activity: A cross-sectional analysis. *Journal of Mathematical Behavior, 39*, 51-66.

[22] Pew Research Center [Pew]. (2015a). Technology device ownership: 2015. Research report: http://www.pewinternet.org/files/2015/10/PI_2015-10-29_device-ownership_FINAL.pdf

[23] Pew Research Center [Pew]. (2015b). Teens, social media, and technology. Research report: http://www.pewinternet.org/2015/04/09/teens-social-media-technology-2015/

[24] Pegrum, M., Oakley, G., & Faulkner, R. (2013). Schools going mobile: A study of the adoption of mobile handheld technologies in Western Australian independent schools. *Australasian Journal of Educational Technology, 29*(1), 66-81.

[25] Pope, H., & Mangram, C. (2015). Wuzzit trouble: The influence of a digital math game on student number sense. *International Journal of Serious Games, 2*(4).

[26] Riconscente, M. M. (2013). Results from a controlled study of the iPad fractions game Motion Math. *Games and Culture, 8*(4), 186-214.

[27] Steffe, L. P. (1992). Schemes of action and operation involving composite units. *Learning and Individual Differences, 4*(3), 259-309.

[28] Steffe, L. P. (1994). Children's multiplying schemes. In G. Harel & J. Confrey (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 3-39). Albany, NY: State University of New York.

[29] Steffe, L. P. (2007). *Problems in mathematics education*. Paper presented for the Senior Scholar Award for the Special Interest Group for Research in Mathematics Education (SIG-RME) at the annual conference of the American Educational Research Association, Chicago, IL.

[30] von Glasersfeld, E. (1995). Sensory experience, abstraction, and teaching. In L. P. Steffe and J. Gale (Eds.) *Constructivism in Education*, (pp. 369-383). Hillsdale, NJ: Lawrence Erlbaum Associates.

[31] Yuan, T., Chae, H. S., Nantwi, G., & Garg, P. (2010). An examination of 300 iPhone math applications: Implications for the development of an educational math game. In *World Conference on Educational Multimedia, Hypermedia and Telecommunications* (Vol. 2010, No. 1, pp. 4083-4088).