The use of technology in teaching and learning the topic of probability and statistics in school mathematics

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Abstract

The use of technology in teaching and learning the topic of probability and statistics in the educational system in our country, Albania, has the potential to be encouraged in different level of classes in the educational system. Lee and Hollebrands (2011) have done better characterization of teaching probability and statistics with the help of technology, through the integration of Statistical Knowledge, Technological Statistical Knowledge, and Technological Pedagogical Statistical Knowledge. Also, we have integrated this framework with Pedagogical Knowledge as a very important factor. In this paper is given as a case study, the process of mastering the concepts of probability and statistics, with students of a lower secondary school in our country to understand the role of technology in the process of teaching and learning, realized concretely during the lessons, specifically throughout a project-based learning by demonstrating the results of the project by the students themselves. Our students recognize and analyze the data, then collect and represent it, to provide graphical presentation Thus, we focus on the potential advantages of using technology in teaching and learning the topics of probability and statistics in school mathematics during different levels of educational process in our country and beyond.

Keywords: *Probability; Statistical knowledge; ICT; Teaching; Pedagogical knowledge.*

1. Introduction

Technology is a part of our daily lives and an integral part of our society. We can state that in Albania about 80 % of students of universities and about 65 % of pupils in high and middle schools are consumers and owners of mobile phones and computers. A small percentage of them, about 35 % use technology as a tool of learning and teaching (data taken by INSTAT, Albania).

In Albania, there have been implemented a series of projects financed by Council of Europe as TEAVET, project of British Council and others, which have the purpose of helping and implementing the idea of teachers and parents, that the process of implementation of ICT in teaching and learning, is a step forward in the preparing of pupils and students for their future. Nowadays, all the teachers, especially those of mathematics, are convinced of the role of technology in the teaching and learning process with many opportunities to do so.

The use of technology makes the teaching process more effective, motivating students in the learning process, giving them even a better feedback.

"When adding technology to mathematical pedagogical activities, it is important for teachers to think about how the use of technology influences representations of mathematics and how the use of technology influences pedagogy" [4].

The information, communication and technology information (ICT) is an integrated part of learning in the working practice during lesson hours in the mathematics courses.

A way of looking at the role of ICT [14] in education in generally and teaching probability and statistics specially, stands for:

1. Information (I) – (or data) in paper or electronic format.
2. Communication (C) – in person or electronically (electronic communications), in writing or voice, telecommunications and broadcasting.
3. Information technology (IT) – including software, hardware and electronics.
4. Communications technology (CT) – including protocols, software and hardware.

Nowadays, the content of the subject of probability and statistics in our country as well as in school curricula around the world, requires that in its beginnings students use technology. So, students in this subject collect data, present them in tables, create graphs, starting from the simplest graphs that are in the form of columns, communicate personally or electronically including applications and interpret data and graphs. In this way, the student develops a special ability in assessing the possibilities and conception of data and a reasonable judgment. They create a statistical evaluation of information and develop their intuition about this data. Also, they can improve their statistical thinking by using instruments in statistics education [12,13].

Information, communication and technology information (ICT) in education is the mode of education that use information and communications technology to support, enhance, and optimize the delivery of information [14, 15].

Worldwide research has shown that an increase in the use of ICT in education with integrating technology to the curriculum:

- can lead to an improved student learning and better teaching methods;
- has a significant and positive impact on students' achievements.

The ICT as technology that supports activities involving information, has a great influence in representing mathematical concepts. The key outputs of educational activities are context, knowledge, experience and products. Such student's activities include gathering, processing, storing and presenting data. Increasingly these activities also involve collaboration and communication between them. So, the students who are continuously exposed to technology through education has better 'knowledge', presentation skills, innovative capabilities, and are ready to take more efforts to learn compared to their counterparts.

In particular, "it is important to determine if representation technology offers, to determine if there is mathematical fidelity, and to consider whether the technology will be used with students as a reinforcement or as a reorganizer" [6].

Like amplifiers, technology makes the most of the actions that can be performed manually, but in a more precise, faster, and more correct way. This raises the question of whether it would be better to act manually or with the help of technology. For example, let's consider that students want to calculate the probability of sides dropping with the sum of digits a odd number when playing with two dices. They can do this by calculating it manually or by constructing a double entry table and counting the favorable cases, because the probability of the event occurring can be calculated in both cases.

As a reorganizer, technology helps change the way students think about a question or a mathematical idea.

A mathematical question is solved in different conditions, whether using a pen and a piece of paper or using technology. Students develop their skills in the use of ICT, express those skills in cases when they research, create and communicate different mathematical ideas and concepts. Using ICT helps them calculate faster results, draw graphs, manage, analyze and interpret various data.

- *First*, the use of technology helps students learn the basic elements of statistical thinking.

- *Second*, it enables students to collect data, observe and consolidate it without difficulty, with little theory and formula.
- *Third*, they create the idea of continuity which is a basic concept in statistics, thus enabling "the use of technology helps to explain modern data models and dialogue" [11].
- *Fourth*, technology encourages interactive learning.

Lee and Hollebrands [4] made efforts to do the combination of the factors below in learning statistics, through exploring of different ways of explaining statistics under the influence of technology and emphasised the importance of pedagogical skills during the incorporation of technology in the process of statistics teaching. Technological Knowledge can help teacher understanding of how technology tools, resources and devices fit into the process of teaching and learning.

These factors are:

- Statistical Knowledge (SK);
- Technological Statistical Knowledge (TSK);
- Pedagogical Knowledge (PK)
- Statistical Pedagogical Technological Knowledge (TPSK).

The connection becomes more intentional for the field of education when a relationship of three factors SK-TSK-TPSK is established simultaneously with Pedagogical Knowledge (PK)- as the philosophical, theoretical, and practical approaches, sets of events, activities, processes, practices, and methodologies that guide teaching and learning [18].

From their graphic presentation it can be clearly stated how they work together and this framework will help the teacher find the best teaching strategies, also understands how technology integration can improve instructional strategies and strengthen content knowledge for learners. Many experiences have shown that statistical skills together with technological statistical skills are used by teachers who have great pedagogical skills in preparing students for statistical subjects.

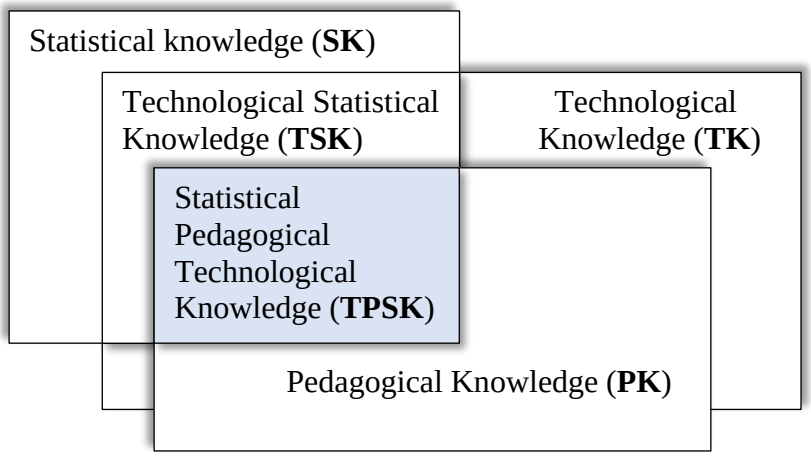

Figure 1. *Framework of Technological Pedagogical Statistical Knowledge*

Within TSK, the focus is on teachers' familiarity with standard technologies in statistics education. It includes familiarity with technological developments such as TinkerPlots (Konold & Miller, 2005), whose dynamic features might support students' learning of statistics [10].

SK- Lee and Hollbrands [4] explain that statistical skills are basic skills and should be combined with their pedagogical skills in teaching and learning satistics. This requires from teachers to be "as active learners and doers of statistical practices" [4].

TSK- has in its focus the process of familiarization of teachers with standards of technology in teaching statistics, which includes adaptation with the application

softwares such as Microsoft Excel Spreadsheet Software, TinkerPlots Software for dynamic data exploration, SPSS, R, etc.

PK- the philosophical, theoretical, and practical approaches, sets of events, activities, processes, practices, and methodologies that guide teaching and learning [18].

TPSK- the first purpose in teaching statistics by the use of technology is the development of TPSK which is a core element of preparing teachers to see the advantage of using technological tools in the process of teaching statistics. If teachers in our schools have good statistical skills and if they know the way of applying the technology in the statistics, being capable pedagogically, they can transmit their knowledge to the students, helping them to create the idea of understanding statistics through technology [10].

2. Research design

All the steps of the process of teaching statistics with the use of technology are illustrated by an example of how teachers of a school explain to their students the process of integration of statistics lessons with the concepts and technological actions.

As a study case in this paper, is the implementation of a project that has been created with two teachers of mathematics and with students of a high school "Imelda Lambertini", Elbasan, Albania. Teachers and students of this school are encouraged using instruments in statistics education, from a study conducted by Prodromou [7] for using TinkerPlots [10], that is a software adaptable for kids of this age in the levels of their education. The teachers tried to explain TinkerPlots as a data visualization and modeling tool and attempted to explore students' use of TinkerPlots during the practice lessons. Through TinkerPlots students manage to create graphics with the data, to interpret them and to attract even conclusions for the presented data [7, 16]. According to their experience in teaching statistics, teachers claim that students have difficulty in mastering the skills of different forms of presentation, to distinguish which graphical representations provide them with relevant information and, above all, the way of reading these graphs.

The purpose of TinkerPlots is to provide a learning environment in which students have the oppurtunity to understand data, draw inferences and develop understanding of statistical concepts [7].

3. TinkerPlots as recommendations of the Curriculum Standards

TinkerPlots is a data visualization and modeling tool developed for use by middle school students through to university students. TinkerPlots can be used to teach grades 4 and up in subjects including math, statistics, social science, or physical or biological science content - in any course in which data are relevant [14,17].

In the USA, TinkerPlots is especially useful for mathematics teachers striving to teach students data analysis in line with recommendations of the NCTM's Curriculum Standards [20], the Common Core State Standards [21] for Probability and Statistics and to inquiry-based science classrooms where students collect and analyze data as part of formulating and testing their own hypotheses. In Australia, TinkerPlots is useful for mathematics teachers striving to teach students the Statistics and Probability strand of The Australian Curriculum: Mathematics [22].

TinkerPlots is similarly useful in most other parts of the world. With the Sampler Engine, students can design and run probability simulations, then plot the results to give a visual representation of the outcomes over many samples. With the Sampler Engine, TinkerPlots expands its focus from data and statistics to incorporate probability.

4. TinkerPlots as simple technological development and pedagogical design

We chose to use Graphing in TinkerPlots [17] for our school project. Below we list some reasons why we chose TinkerPlots, as well as to suggest it further as a *simple technological development and pedagogical design* in teaching and learning [17].

Teachers start where their students are:	Students can begin using TinkerPlots without knowledge of conventional graphs or different data types, without thinking in terms of variables or axes.
By progressively organizing their data:	By ordering, stacking, and separating data icons, students gradually organize data to answer their questions. Students can analyze data that come with the program, that they download from the Internet, or that they enter themselves.
Students design their own plots:	Using the construction set of basic operations, students create a wide variety of graphs, including standards like pie charts, histograms, and scatterplots, and novel graphs of their own invention.
Transforming one display into another:	Because plots are built up in stages, students can deconstruct unfamiliar plots to learn how to interpret them. Students can save the current plot configuration as a new command ("skyline graph") to later recreate that plot type in one step.
In search of group differences and trends:	To perceive variability in data, TinkerPlots offers more than position along axes; it also offers differences in icon size, color, and sound. These additional modalities allow students to detect co-variation in powerful and intuitive ways [17].

5. Project-based learning

The following material is a case of a project conducted by students of grades 6 - 9 (95 by 114 students in total) of this school related to oral hygiene. Below, through concrete examples from graphic representations, we can explain that TPSK is right to learn statistics, by studying how these teachers of this school can use their pedagogical knowledge to better combine the use of software with concepts of statistics. In this case, the presentation of the results of this project was done using TinkerPlots, unlike other times when students were used to seeing demonstrations using ScaterPlots. Through this project they can:

- create a statistical study with students of their schools.
- show how to improve hygiene and the importance of oral health.
- motivate students for individual or group work, encouraging them in study work based on their desires and skills in using technology.

Students have collected data related to this topic, but we will present only a part of relations that exist among these data.

- The first care for oral health is brushing - How many times a day do you wash your mouth?
- The second comes how long it takes them to clean their mouth - How long do you need to wash your mouth?
- One of the factors that influences the oral health is the visit to the dentist - How many times do you go to your dentist in 6 months?

For implementing this project, students make its application in different classes. First, they complete the questionnaire only by sixth graders. Then, they complete the questionnaire by students of other classes. The range of sample is expanded to one or two classes.

The principal purpose is to make them known that:

- The larger the sample, the better the population is represented and the more realistic the answer to the truth.

In this project, created and implemented by students, before they manage to represent and use data, they should be able to know sufficiently theoretical statistical concepts such as: median, mode, amplitude, ways of comparing the samples and to see the difference of these data among different classes of population. Being assisted by teachers, students collect data for each individual and present them in a table. The teacher requires them, with Tinker Plots, and as the representation of data should be done in colons. All data for each student must be collected in a common spreadsheet that is transferred to TinkerPlots directly from Microsoft Excel. Students should be guided by teachers in researching validity evidence for this instruments in their statistical education [13].
With the help of teachers, students are able to create graphs of the dependency between two variables for different samples. In various graphical representations with TinkerPlots, students can see a demonstrated presentation of data in two oriented axes, according to the relationship of the sample sizes to each other. Using the software enables different graphic creation, helping to compare different layouts for different levels of choices offered (one class, two classes and so on). Step by step, utilizing their ability and desire to use technology, teachers see the development of their statistical thinking and the beginnings of changing their education through the use of technology in their group or individual activation during the educational process throughout the project [12].
Also, a specialty of using TinkerPlots, is the fact that, with sample expansion, better results can be obtained. In this way, students can see themselves by this instrument the effect of changing the sample size and compare the results obtained in the changed graph [12,13]. Using a sequence of "growing champion" activities with the help of Tinker Plots is a pedagogical design that can help students visualize how larger samples better represent the population [7].
The teacher first tells the students that the data is presented on small cards with selected attributes.

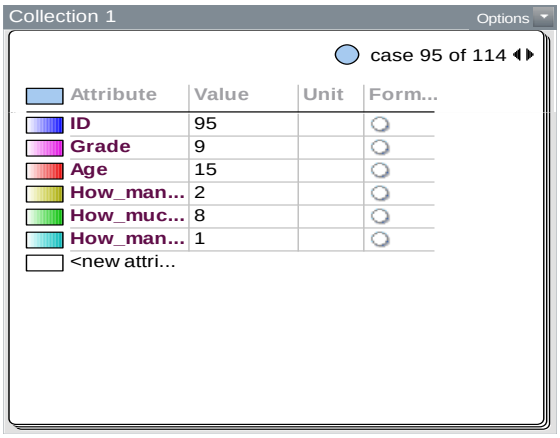

Figure 2: *An example of single TinkerPlots data card*

The next step is to combine information of these single cards into one spreadsheet.

Table 3: *a spreadsheet representing all data cards*

Collection 1 — Options

	ID	Grade	Age	How_m...	How_m...	How_m...
76	76	8	14	3	5	1
77	77	8	14	2	5	2
78	78	8	14	3	3	1
79	79	8	14	2	6	1
80	80	8	14	1	7	1
81	81	8	14	3	6	1
82	82	8	14	2	3	1
83	83	8	14	2	5	1
84	84	8	14	2	5	1
85	85	8	13	3	6	1
86	86	9	14	2	2	3
87	87	9	15	2	4	2
88	88	9	15	2	4	1
89	89	9	15	3	5	1
90	90	9	15	3	1	1
91	91	9	14	3	6	1
92	92	9	14	2	6	1
93	93	9	15	2	5	1
94	94	9	14	2	4	1
95	95	9	15	2	8	1
96	96	9	15	2	3	2
97	97	9	14	2	6	1

With this information teachers show their students how different graphics can be created by software. For example, TinkerPlots can be created for mouthwash duration, for sixth graders, sixth and seventh graders, and so on.

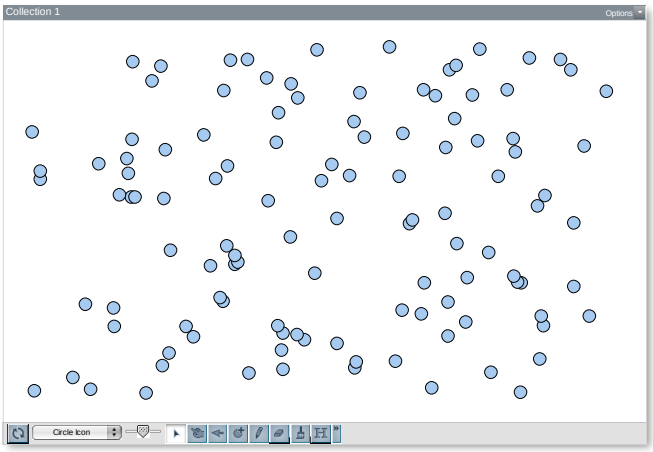

Figure 4: *TinkerPlots of data*

Different data can be combined in such a way as to give students the opportunity to observe different images. So, the statistical thinking and interpretation is developed by this study.

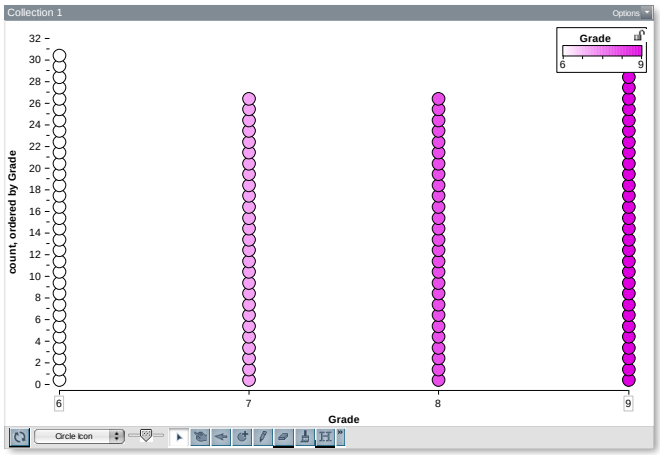

Figure 5: *Another form of data representation*

From the graphic representations students can see that with the increase of the sample volume the result also changes, getting closer and closer to the expected result.

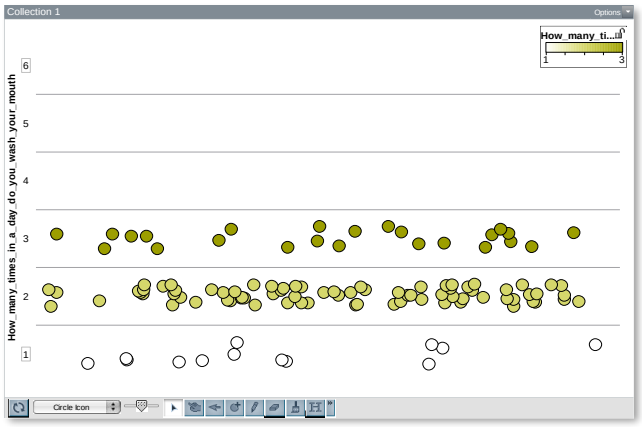

Figure 6: *Times of brushings for day*

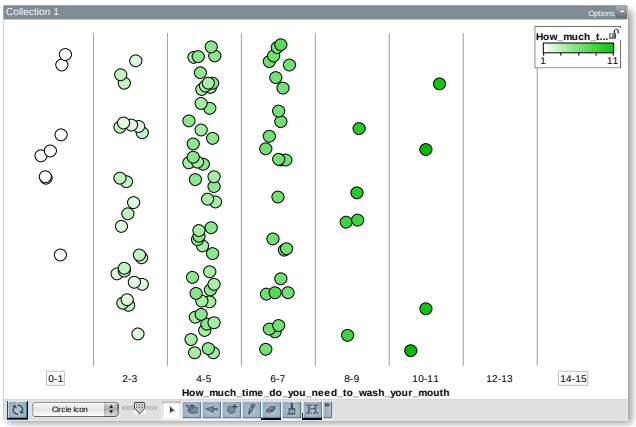

Figure 7: *Time needed for brushing teeth*

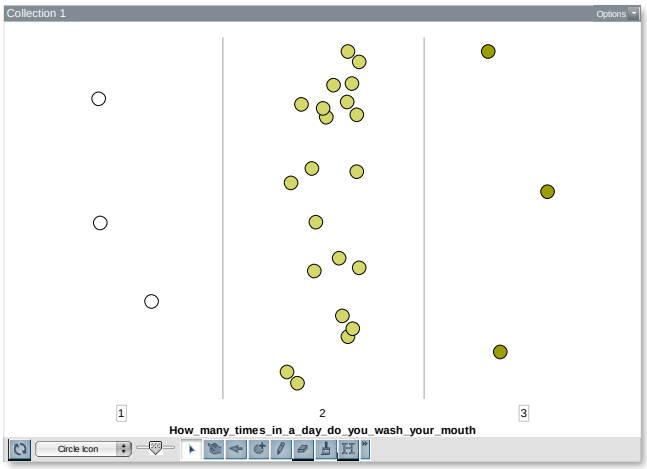

Figure 8: *Number of brushings for day*

The graph below shows how TinkerPlots allows teachers to present data in a coordinate system of axes, named according to the variables used. For example, the graph below shows how the correlation between the number of brushings and the duration of brushing affects oral health.

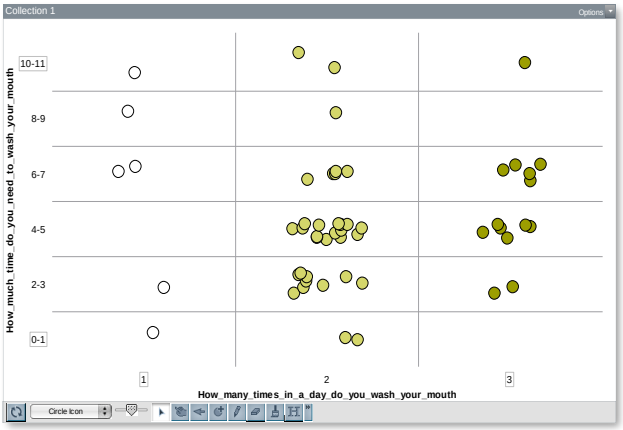

Figure 9: *Relationship between the number of brushings and the duration of brushing*

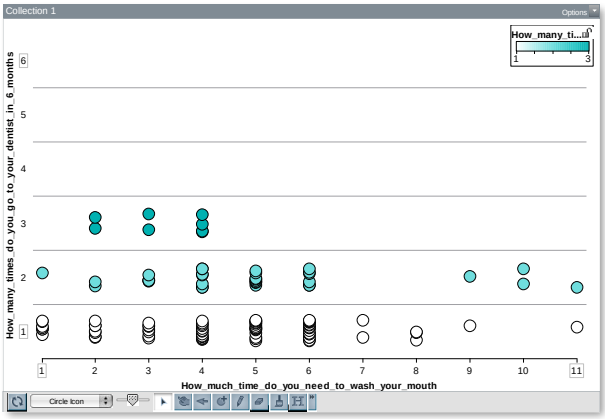

Figure 10: *data generated from grade 6 & 7*

Also, the relationship between the facts "how often we visit the dentist and the duration of the brushings" is given in the graph below.

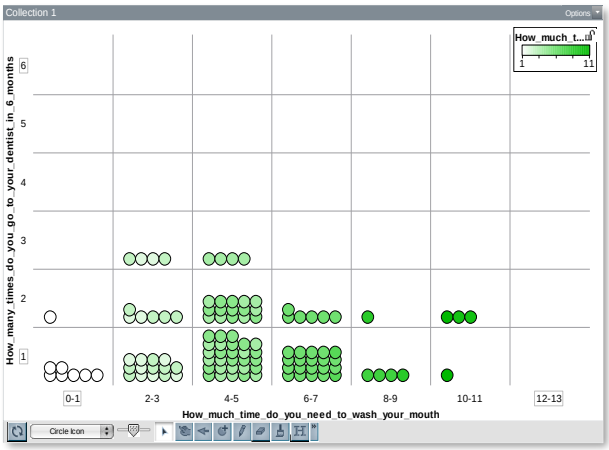

Figure 11: *relationship between frequency of dentist visits and duration of brushings*

TinkerPlots helps learners see the difference in results by changing the size of a sample by increasing or decreasing the number of classes in the graphical presentation. So, the teacher gives them the opportunity, for a sample of a certain size, to get a fixed result and offer a consistent model. In this way, students reinforce theoretical concepts such as: data collection by designing a questionnaire for TinkerPlots, using, presenting and reading graphs from this data, interpreting, as well as comparing two samples, their amplitudes etc.

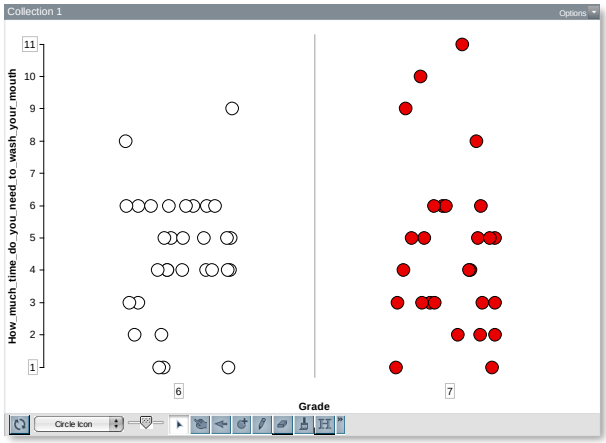

Figure 12: *data generated from grade 6 & 7*

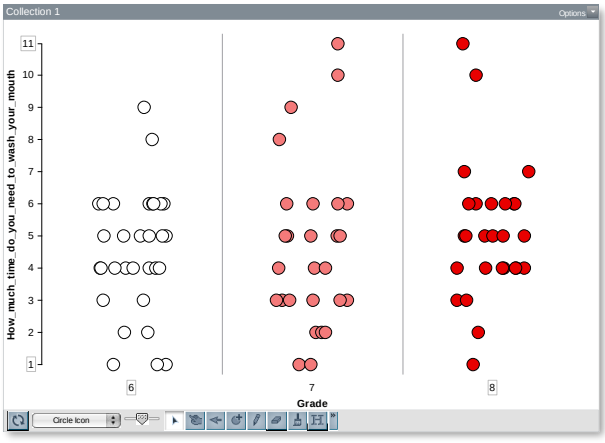

Figure 13: *data generated from grade 6, 7 & 8*

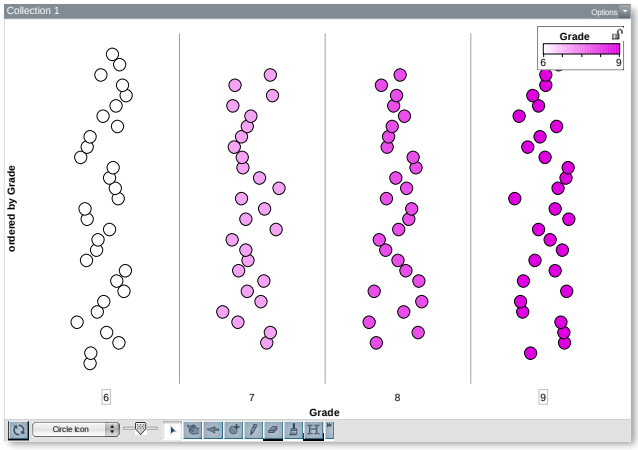

Figure 14: *data generated from grade 6, 7, 8 & 9*

Taking a look at the management of TPSK, we can say that while working with Tinker Plots, students gain skills in determining and changing the volume of a selected choice and how important this change is in representing a population presentation. Teachers also apply TPSK to these concepts.

6. Discussion

The purpose of this paper is to show the use and need of TPSK in teaching statistics. According to our information, the application of TinkerPlots has not been used before in teaching probability and statistics in the pre-university education schools of our city, so we have implemented it for the first time in this school of this level during a school project in this subject. We think it would be interesting for math teachers to use this application as a teaching technique and to train students to practice this use during teaching activities in the subject of statistics in more schools. Students participating in the process were enthusiastic about using this software to present, interpret and read data, which was easier than the cases of other techniques.

It should be noted that the project that is being practiced by us has been implemented by students of a non-public school. We emphasize that the school has high level laboratories offering students the opportunity to teach internships in all subjects. In these schools, there are intelligent whiteboards in every classroom, which are tools to help teachers and students in teaching and learning, but also a good opportunity to explain how to make presentations with TinkerPlots [16]. However, we think that a similar project and other project-based learning can be easily realized with technological laboratories in public schools which are used by students during practical activities in different subjects.

From this project, we can see that the use of software helps and strengthens the understanding of statistical skills with the help of technology.

7. Conclusions

Already today, the desire and ability of students to use computers and other tools/ electronic devices is well known. Mathematics teachers try to attract students to activities where the use of ICT during math classes, specifically the topics of probability and statistics, motivates them for individual or group work. They think that increasing the use of ICT in education with integrating technology to the curriculum has a significant and positive impact on students' achievements. The use of software in the daily teaching of statistics is a powerful tool for a better learning process, from a theoretical and practical point of view.

A review of the mathematics curricula in pre-university education, specifically the topic of statistics and probability make it possible to create space for the concrete treatment and application of

technology with applied techniques and tools as simple technological development and pedagogical design for teaching and learning.

Thus, by creating these conditions, teachers will find more space and opportunities *to focus on:*

- students' ability to use the computer, other tools/ electronic devices and the concrete use that ICT finds during math classes, especially during project-based learning;
- applied techniques for data visualization and modeling tool developed for use by students as recommendations of the Curriculum Standards;
- applications software as simple technological development and pedagogical design for teaching, learning statistical thinking and acting using efficient instruments.
- the effect of using technology in changing learning and developing thinking in education through practice.

Finally, we thank the teachers and students of this school for their practical skills and support the initiative for further improvement of this project with the help of ICT.

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