A Maplet for Analyzing Progressive Casino Game Betting Systems

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Abstract

This work is primarily the product of the first listed author, who completed it as a student under the direction of the second and third listed authors. In this paper we analyze three progressive betting strategies, each applied to three casino games. The betting strategies considered are Martingale, Paroli, and Fibonacci, each applied to the casino games blackjack, craps, and roulette. The aim of this analysis is to try to identify optimal betting strategies for each game with a given or maximum number of bets placed that pay 1:1. The purpose of this work is not to try to discover methods for "beating the house," which are already known to not exist, but rather to search for methods for advancing gameplay through a given or maximum number of bets while retaining the possibility of earning a profit. Programming in the computer algebra system Maple, more specifically through the use of a Maplet, which is like an applet but uses (and requires) Maple's engine, will be used for calculations.

1   Introduction

Casino gamblers have been attempting to "beat the house" ever since organized gambling was introduced during the Chinese Tang dynasty between the 7th and 10th centuries A.D. Players have been attempting to increase their odds of winning through both legal and illegal means, using playing strategies such as choosing when to make a particular decision or how much to wager on a specific outcome. The motivation for this paper came from a run of bad luck by one of the authors at a casino over several months. The hope was that by using some mathematical analysis, we would be able to see how to use betting strategies to skew outcomes in our favor over the short or long term.

Please note that we do not wish to give the impression that anything can be done to change the odds to players or the house. Rather, we only wish to try to identify betting strategies that allow players to limit risk while retaining the potential for success. With a focus on games with around a 50% probability of success, or, as it is known, "1:1 (one-to-one) odds," by applying the betting systems, we propose that players could play a chosen number of bets with only a minimal loss or even a potential profit. The three games we will play through simulation are blackjack, craps, and roulette, to which we will apply the three betting systems Martingale, Paroli, and Fibonacci.

2   The Games

2.1   Blackjack

The most common theory for the origin of blackjack gives credit to early 18th century France with the invention of the game *Vingt-et-Un*, which translates from French as *Twenty-One*. Under the reign of King Louis XV, the game was played at French Royal Court. By the early 19th century it had made its way onto the streets of New Orleans, though it was played with a slight variation on the rules as compared to today. When gambling became legalized in Nevada during the 20th century, the game began being hosted by casinos and gambling halls. Its name was changed to *blackjack* with the hope of growing its popularity, and additional payouts were given to players who won with a black Jack (the Jacks of clubs or spades) or the Ace of spades. These bonus payouts eventually became less common after the game did indeed grow in popularity [3].

A few variations exist for the rules of blackjack, although the odds of winning change only very slightly. The standard rules for blackjack and the parameters for our analysis are:

1. All cards are assigned point values. Cards numbered 2–10 are assigned the number of points of their face value, while Kings, Queens, and Jacks are assigned 10 points. Aces can be assigned either 1 or 11 points, as chosen on each deal by the player.

2. The dealer deals the player two cards, while the dealer receives two cards with one facing up. The player then decides whether to "hit," meaning to take an additional card, or "stand,"

meaning to not take an additional card. If the player hits, then they decide again whether to hit or stand, continuing in this manner until they either stand or "bust," meaning they have more than 21 total points. The player loses if they bust. If the player stands with fewer than 21 points, then the dealer repeatedly draws cards until they either beat the player or bust. In this scenario the house loses only if the dealer busts.

3. For our analysis, we will assume eight full decks of cards are used. Also, if the player's first two cards are a "blackjack," meaning an Ace and any other card worth 10 points for a total of 21 points on the deal, the payoff to the player is 3:2, meaning any bet earns a profit of 50%. Finally, we will assume the player will follow the basic strategy of the betting systems analyzed, which dictate gameplay based on scenarios. Although electronics cannot be used at blackjack tables, basic strategies can be printed and used at tables legally.

Following these rules, the odds are in the dealer's favor, but only by a margin of 0.43096% [5]. That is, the dealer wins on average 50.21548% of the time, while the player wins on average 49.78452% of the time.

2.2   Craps

The most common theory for the origin of the dice game craps is that it was invented during the Crusades, when it was known as *Hazzard*, and grew in popularity during the gambling boom in 17th century France. As with blackjack, craps came to the United States via New Orleans as a street game called *street craps*. The term *craps* comes from the French word *crapaud*, meaning *toad*, which people tended to resemble as they crouched over to play the game on a floor or sidewalk. The American version of craps saw a few minor rule changes over the years, but the game has basically been played in a very similar manner for many years.

Craps consists of 27 bets that can be placed in prediction of the sum of two dice rolled by a *shooter* [4]. A few bets are inverses of each other, so playing both during the same roll would be illogical, but any other combination of bets is reasonable. The parameters for our analysis are:

1. The player only plays the *Field* bet, meaning the sums 3, 4, 9, 10, and 11 are 1:1 winners, while the sums 2 and 12 are 2:1 winners.

The odds for the Field craps bet are in the house's favor by a margin of 5.56% [4].

2.3   Roulette

Roulette is a game played with a small white ball, called the *pellet*, spinning around a horizontal wheel, on which it eventually settles into one of the following 37 or 38 numbered positions: ones

labeled 1–36 and split between red and black with 18 positions for each, one labeled 0, and, sometimes, also one labeled 00. The roulette wheel was invented by the legendary French mathematician Blaise Pascal when he was attempting to create a continuous motion mechanism requiring no outside force to stay in motion [6]. While Pascal's intended experiment failed, he nonetheless accidentally succeeded in creating the roulette wheel.

As with craps, there are many different ways a player can bet on roulette, including on individual numbers, splitting a bet between two or four numbers, and dividing the table into thirds by betting on 1–12, 13–24, or 25–36. The parameters for our analysis are:

1. The player makes the same even-money (1:1) allowed bet on each spin. One allowed bet is "even/odd," in which a player choosing even wins if the pellet lands on a nonzero even number, and a player choosing odd wins if it lands on an odd number. Another allowed bet is "high/low," in which a player choosing high wins if the pellet lands on a number in the range 1–18, and a player choosing low wins if it lands on a number in the range 19–36. The final allowed bet is "black/red," in which a player wins if and only if the pellet lands on their chosen color.

2. The wheel has 38 spaces, including both 0 and 00, since this is the type of wheel most common today. With this wheel, all of our allowed bets have a probability of winning of $\frac{18}{38}$.

The odds for all our allowed bets are in the house's favor by a margin of 5.26% [5].

3 The Betting Systems

3.1 Martingale

The Martingale betting system, which was introduced by French mathematician Paul Pierre Lévy in the 18th century, is likely the most common progressive betting system used today. Legend has it that the system derives its name (with one letter changed, for some reason) from the last name of early 18th century London casino owner John H. Martindale, who encouraged patrons to use it in his own casino, which led, ironically, to its bankruptcy. The system has transcended standard casino gambling, and is now used in many nontraditional forms of gambling, such as investment banking, stock brokering, and sports betting. The rules of Martingale are straightforward—with each loss, one doubles their bet, until they win. Although starting bet sizes of 5% of one's bankroll are more common in practice, we will use 1% in our analysis in order to more clearly see long-term trends [2].

Table 1 shows an example of the Martingale system in practice, with a beginning bankroll of $100 and initial bet size of $1. Under Martingale with a beginning bankroll of $100, after six consecutive losses it would be not possible to make a seventh bet. Nonetheless, even though Martingale is a high risk system, it is also true that for one whose beginning bankroll is sufficient to fund the next bet after n consecutive losses, a win on that next bet would return the beginning bankroll plus the amount of the

initial bet. Exhausting one's bankroll is not the only drawback to the system though. Casinos often have betting limits, so one might reach a point where doubling after a loss would not be allowed.

Table 1: Example of the Martingale betting system.

Round	Bet	Win or Loss	Bankroll
			$100
1	$1	Loss	$ 99
2	$2	Loss	$ 97
3	$4	Loss	$ 93
4	$8	Win	$101
5	$1	Win	$102

Table 2 shows the results of applying 1000 trials of the Martingale betting system to each game, stopping if 25, 50, or 100 rounds are reached in each trial, with a beginning bankroll of $500 and initial bet size of $5 for each trial. Within each cell of the table, the first number is the average number of rounds played before either the number of rounds indicated in the first column was reached or the next bet could not be made, and the second number is the average final bankroll. As we can see from the table, blackjack shows the highest likelihood of success, with almost 2% in expected profit with at most 25 rounds, and around 0.2% in expected loss with at most 100 rounds.

Table 2: Results from trials of the Martingale betting system.

Rounds	Blackjack	Craps	Roulette
25	23.59 / $509.10	22.76 / $482.60	23.33 / $502.25
50	42.95 / $504.99	40.96 / $473.82	40.56 / $475.34
100	72.56 / $498.88	65.69 / $442.77	66.99 / $441.38

We will now demonstrate a Maplet[1] entitled **Casino** written by the authors which can be used to analyze the three betting systems and three games considered in this paper. This Maplet is available for download at the link labeled [S1] in Section 5. The code that underlies the Maplet is unique to Maple, but could easily be altered for any programming language and any game as long as the player knows the house edge, probability of winning a round, and maximum number of rounds to be played. The **Casino** Maplet allows users to select the betting system and game to be analyzed, and enter the number of rounds, trials, initial bet amount, and beginning bankroll. Clicking the **Compute Parameters and Graph Distribution** button then causes the mean number of turns, mean ending money value, largest ending money value, and smallest ending money value to be displayed, as well as a graph showing the distribution of ending money values.

The screenshots of the **Casino** Maplet in Figures 1–3 show the distribution of ending money values for 1000 trials of the Martingale betting system for all three games with at most 25 rounds, a beginning bankroll of $500, and an initial bet size of $5. All show a bimodal distribution with left skewness and a majority of trials ending in profit.

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[1] A Maplet is like an applet, but uses the computer algebra system Maple, and is written using Maple functions and syntax.
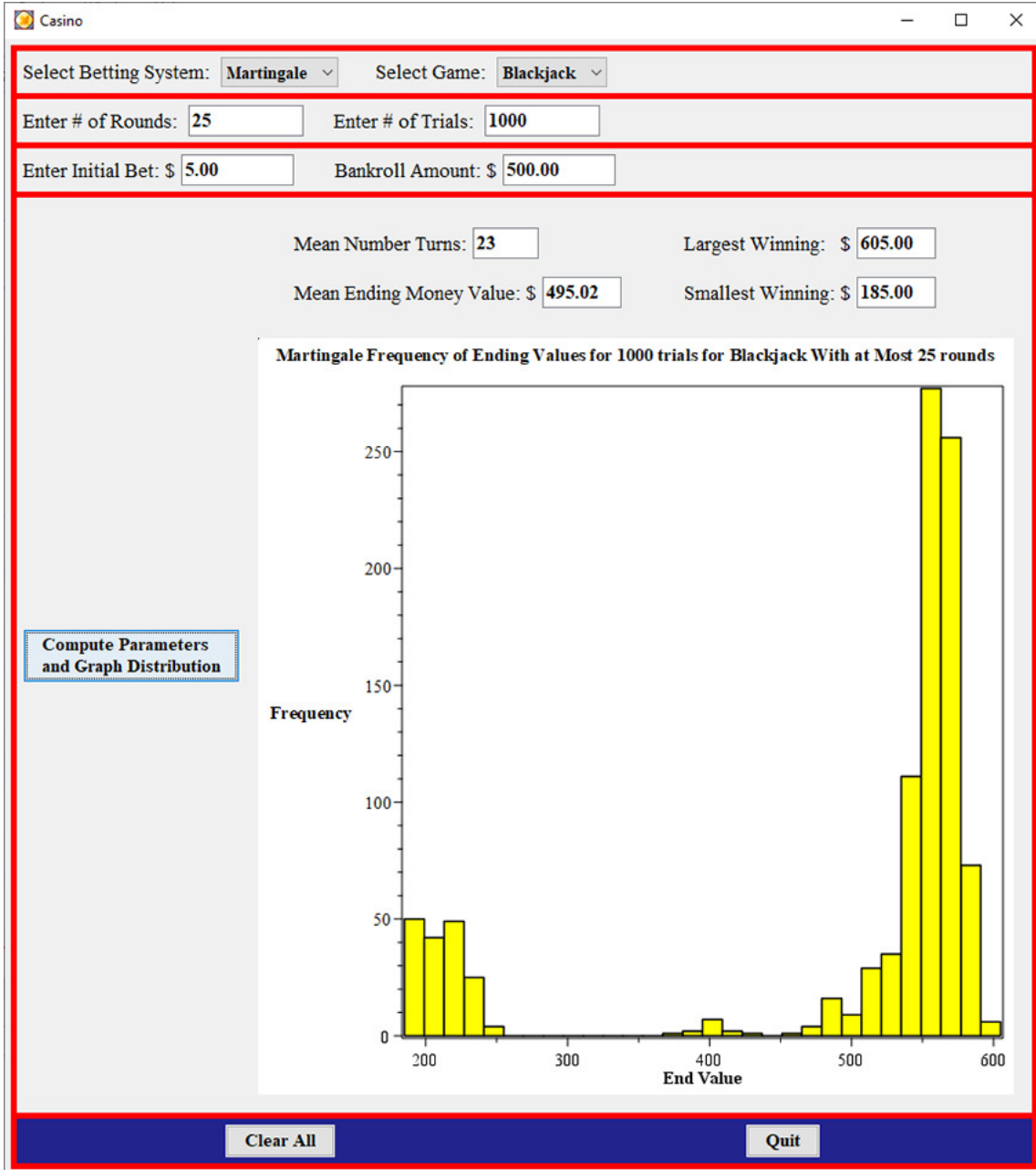
Figure 1: Ending Money Values for Blackjack With at Most 25 Rounds Under Martingale System
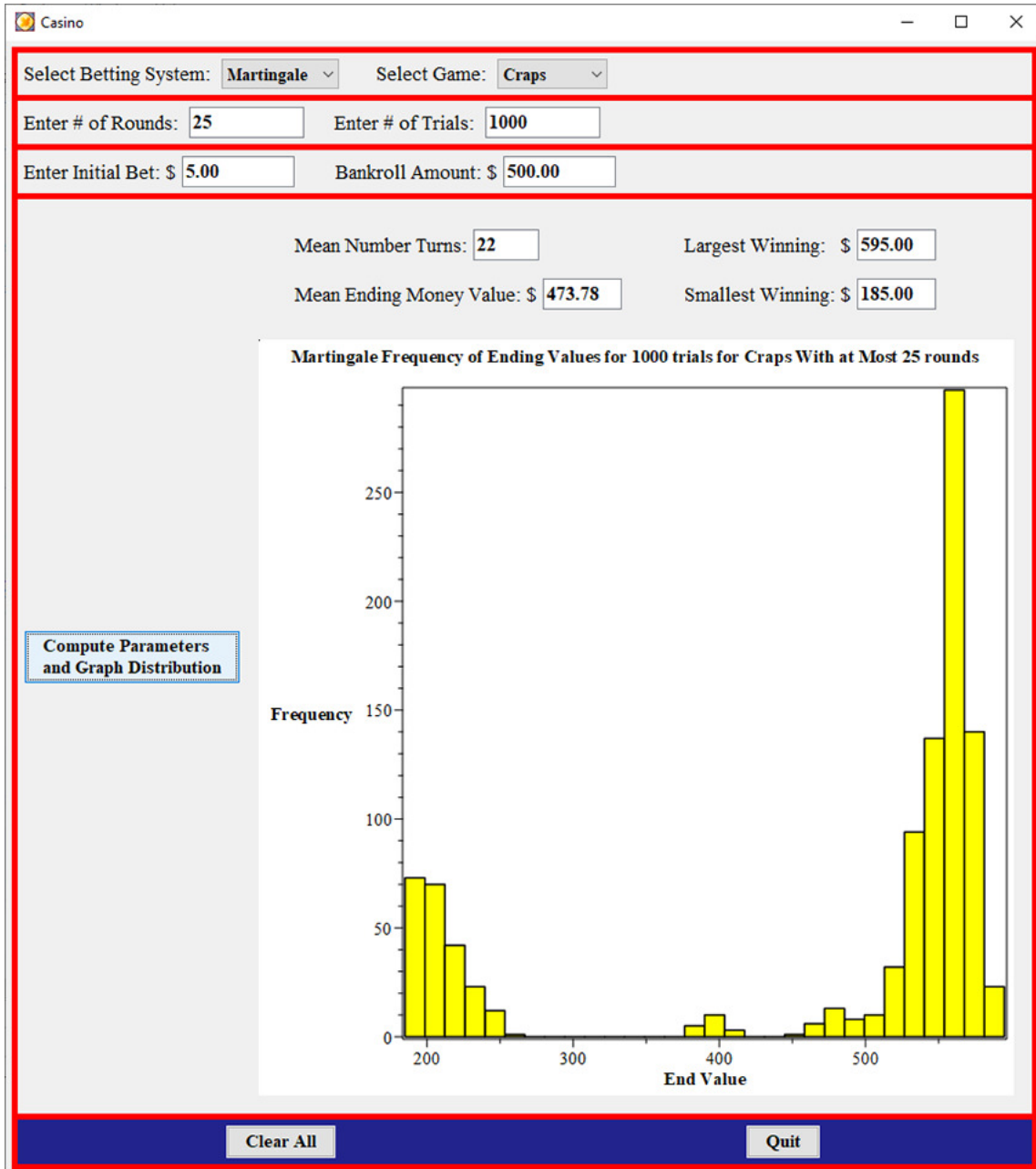
Figure 2: Ending Money Values for Craps With at Most 25 Rounds Under Martingale System
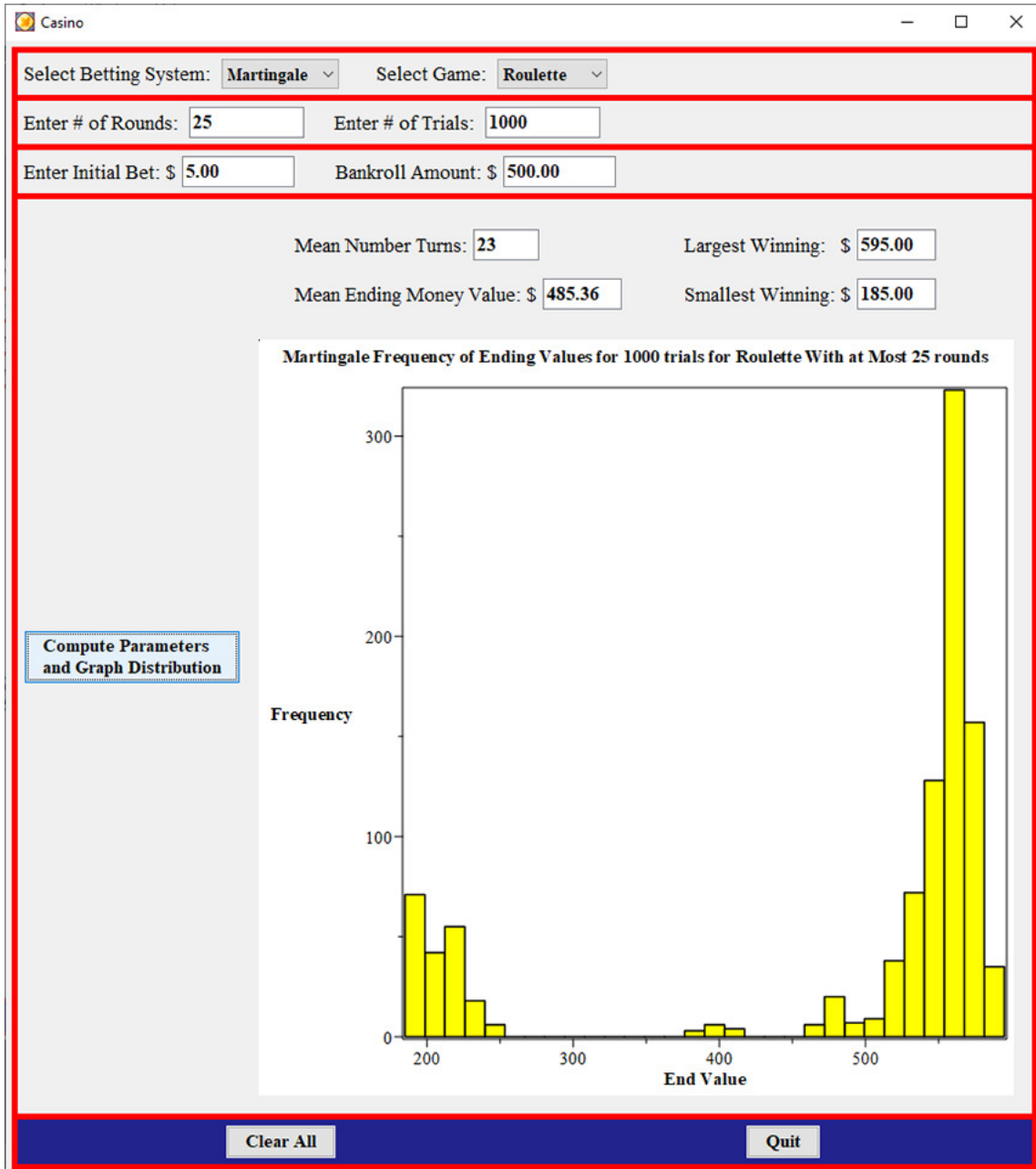
Figure 3: Ending Money Values for Roulette With at Most 25 Rounds Under Martingale System

3.2   Paroli

The Paroli betting system derives its name from the Latin word *par*, which means *to break even*. Like the roulette wheel, the Paroli betting system has been credited as an invention of French mathematician Blaise Pascal. While the Martingale betting system is a high-risk system, Paroli, also known as "anti-Martingale," allows players to set a maximum amount they are willing to lose with each play [1]. Under Paroli, players double their bet after each round until either three consecutive wins or a loss, after which they return to their initial bet amount [5].

Table 3 shows an example of the Paroli system in practice, with a beginning bankroll of $100 and initial bet size of $1. Under Paroli with an initial bet size of $1, three consecutive wins results in a profit of $7, or, equivalently, "pays 7 to 1." Since each round offers a chance of winning of approximately $\frac{1}{2}$, the probability of winning three consecutive rounds is about $\frac{1}{2^3} = \frac{1}{8}$. That is, under Paroli, players are taking a chance on earning a profit of 7 to 1 against odds of winning of 8 to 1.

Table 3: Example of the Paroli betting system.

Round	Bet	Win or Loss	Bankroll
			$100
1	$1	Win	$101
2	$2	Win	$103
3	$4	Win	$107
4	$1	Loss	$106
5	$1	Win	$107

Table 4 shows the results of applying 1000 trials of the Paroli betting system to each game, stopping when 25, 50, or 100 rounds are reached in each trial, with a beginning bankroll of $500 and initial bet size of $5 for each trial. The number in each cell of the table is the average final bankroll. In comparison with the Martingale system, the Paroli system rarely returns a large profit. However, under Paroli players can guarantee playing any specified number of rounds, since the most they could lose on any single round would be the value of their initial bet. For example, if a player wanted to guarantee being able to play 25 rounds, they could start by betting 4% of their bankroll, since this would require 25 consecutive losses for their entire bankroll to be lost.

Table 4: Results from trials of the Paroli betting system.

Rounds	Blackjack	Craps	Roulette
25	$501.79	$488.11	$489.81
50	$500.77	$476.43	$478.01
100	$495.32	$455.11	$455.39

The screenshots of the **Casino** Maplet in Figures 4–6 show the distribution of ending money values for 1000 trials of the Paroli betting system for all three games with 100 rounds, a beginning bankroll of $500, and an initial bet size of $5. All show a more normal distribution than Martingale, with more predictability and less volatility. In particular, the ending money values rarely dip below $300.
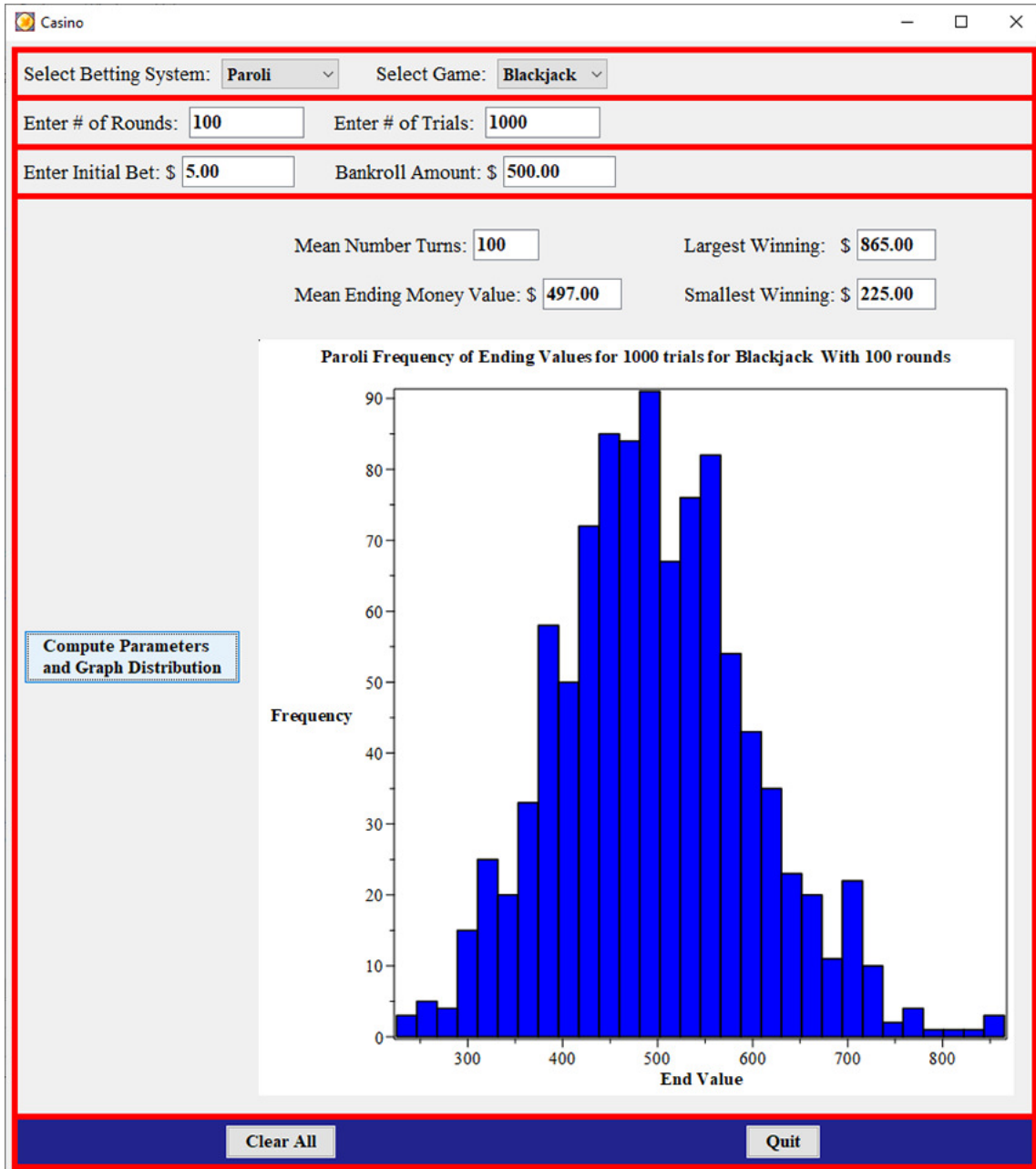
Figure 4: Ending Money Values for Blackjack With at Most 100 Rounds Under Paroli System
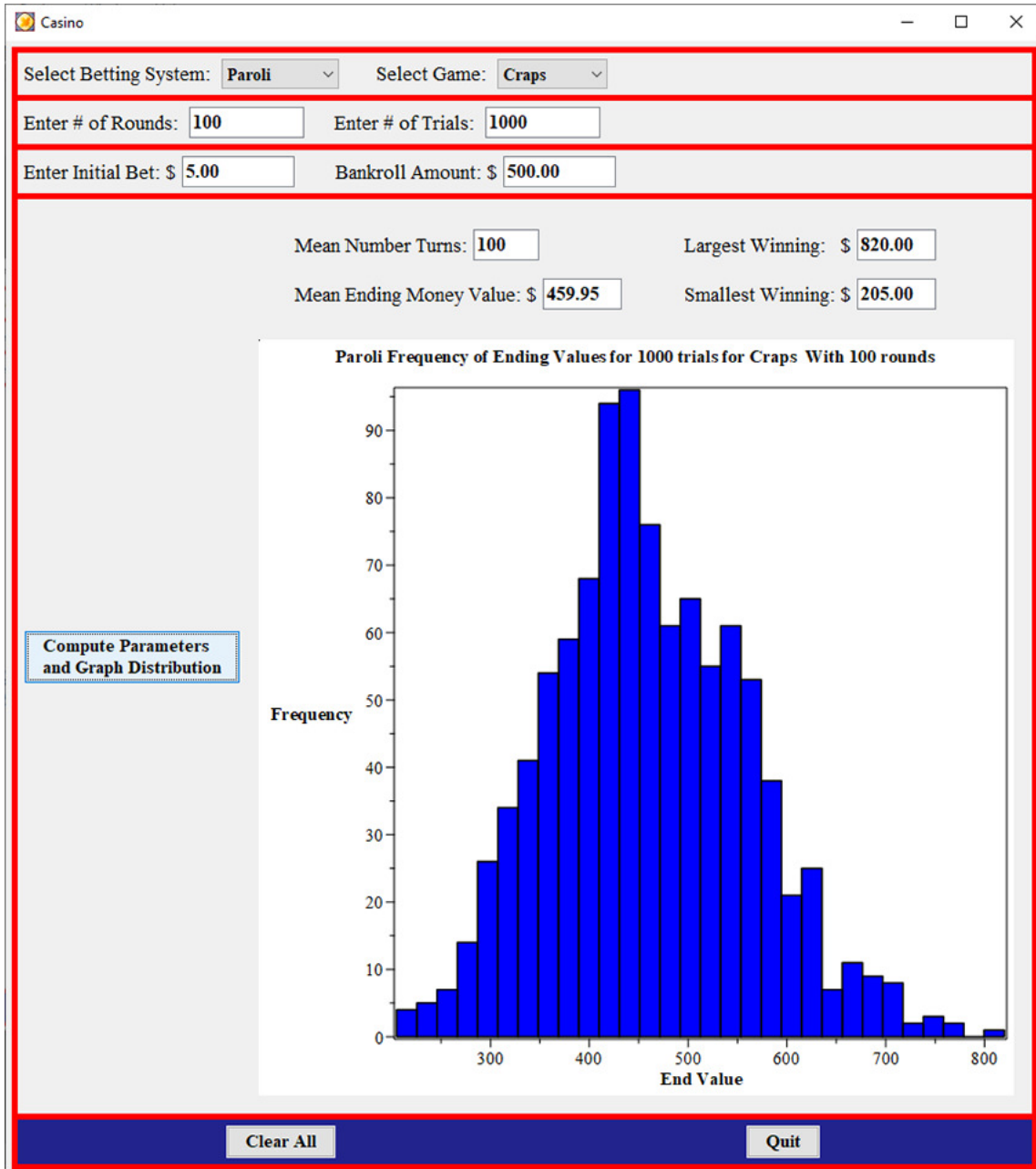
Figure 5: Ending Money Values for Craps With at Most 100 Rounds Under Paroli System
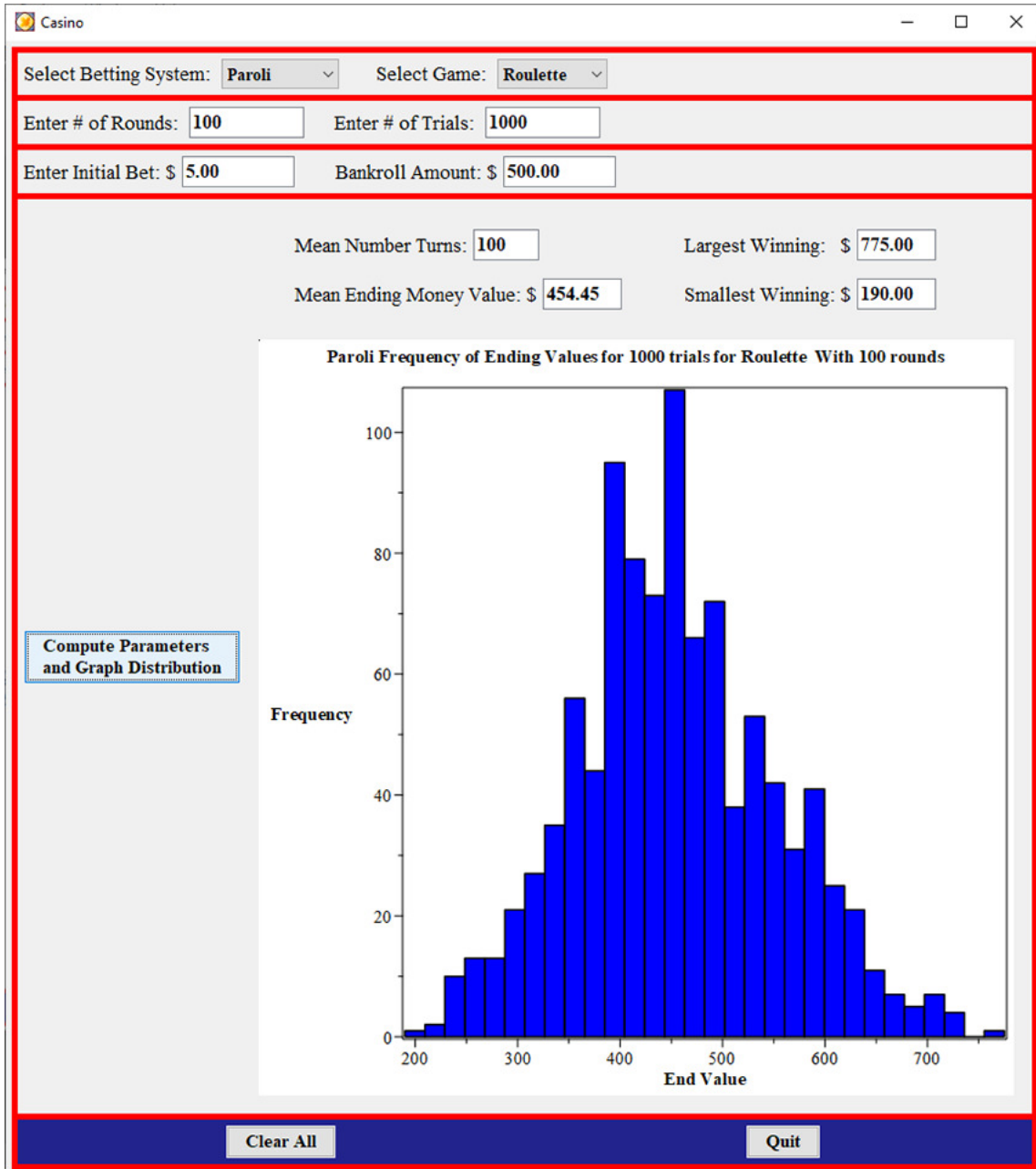
Figure 6: Ending Money Values for Roulette With at Most 100 Rounds Under Paroli System

3.3 Fibonacci

Recall that the Fibonacci number sequence begins with the terms $F_0 = 0$ and $F_1 = 1$, with subsequent terms F_n for $n > 1$ being the sum of the previous two terms: $F_n = F_{n-1} + F_{n-2}$. That is, the Fibonacci sequence is: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ... In the Fibonacci betting system, each bet made by a player is their initial bet scaled by a multiplier from this sequence. For the first bet, players multiplying their initial bet by the third number in the sequence, $F_2 = 1$. After each loss, the bet multiplier moves forward one position in the sequence, from F_i to F_{i+1}. After each win, the bet multiplier moves back two positions in the sequence, from F_j to F_{j-2}, though never moving past the third number in the sequence [5].

Table 5 shows an example of the Fibonacci system in practice, with a beginning bankroll of $100 and initial bet size of $1. A drawback to the Fibonacci system is that several consecutive losses can quickly lead to very large bets. For this reason, literature on the system often recommends that players use an initial bet size of between only 1% and 5% of their beginning bankroll.

Table 5: Example of the Fibonacci betting system.

Round	Bet	Win or Loss	Bankroll
			$100
1	$1	Loss	$ 99
2	$2	Loss	$ 97
3	$3	Loss	$ 94
4	$5	Win	$ 99
5	$2	Win	$101

Table 6 shows the results of applying 1000 trials of the Fibonacci betting system to each game, stopping if 25, 50, or 100 rounds are reached in each trial, with a beginning bankroll of $500 and initial bet size of $5 for each trial. Within each cell of the table, the first number is the average number of rounds played before either the number of rounds indicated in the first column was reached or the next bet could not be made, and the second number is the average final bankroll. The results are similar to what we saw for the Martingale and Paroli systems, with even with a better chance of a higher profit. However, unlike Paroli, Fibonacci does not allow players to guarantee being able to play any specified number of rounds, and could also allow bets to become very large.

Table 6: Results from trials of the Fibonacci betting system.

Rounds	Blackjack	Craps	Roulette
25	24.73 / $504.47	24.54 / $496.19	24.41 / $485.67
50	47.49 / $513.69	46.32 / $467.82	46.67 / $479.24
100	90.22 / $530.51	85.08 / $450.14	86.13 / $461.16

The screenshots of the **Casino** Maplet in Figures 7–9 show the distribution of ending money values for 1000 trials of the Fibonacci betting system for all three games with 50 rounds, a beginning bankroll of $500, and an initial bet size of $5.
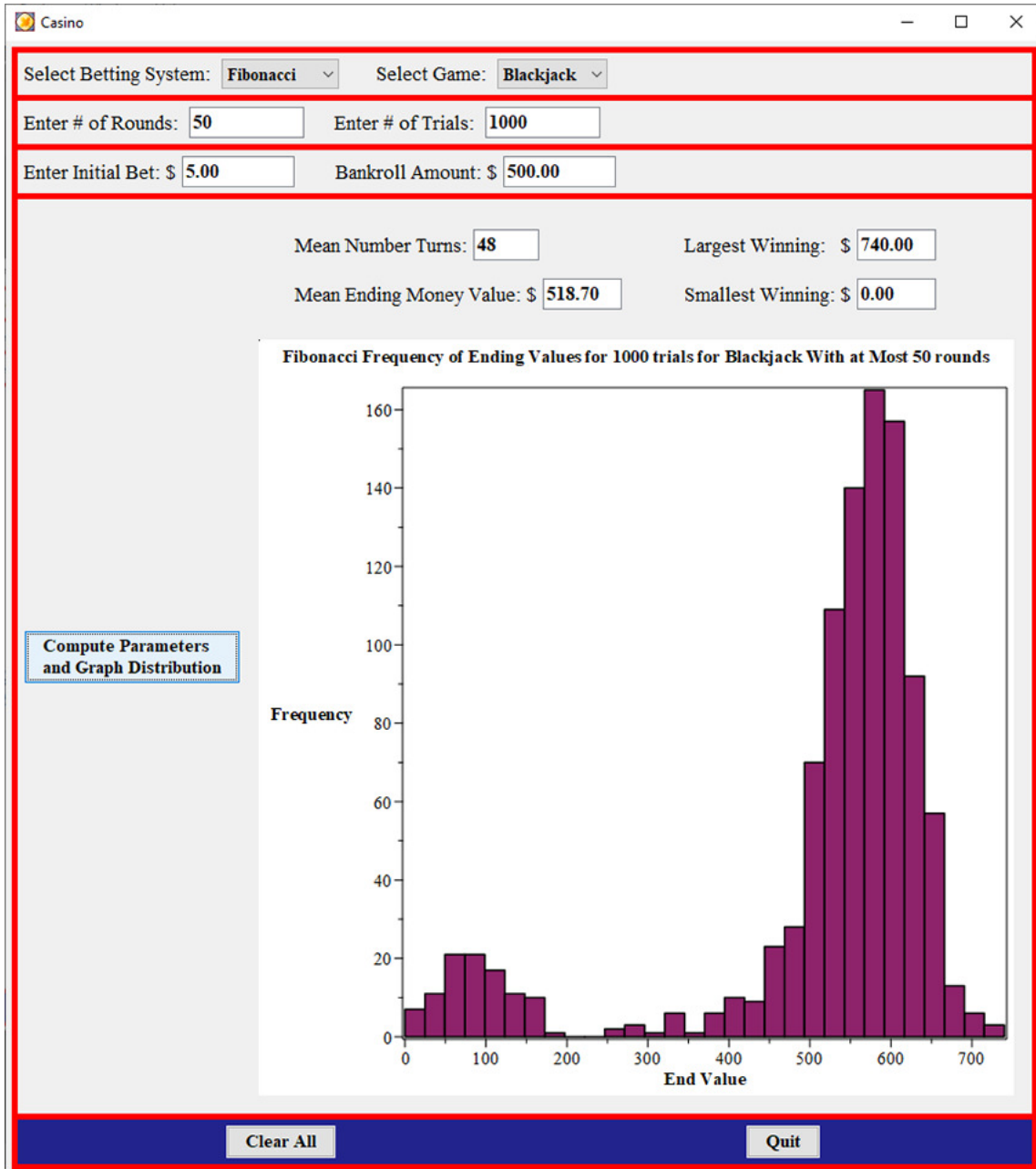
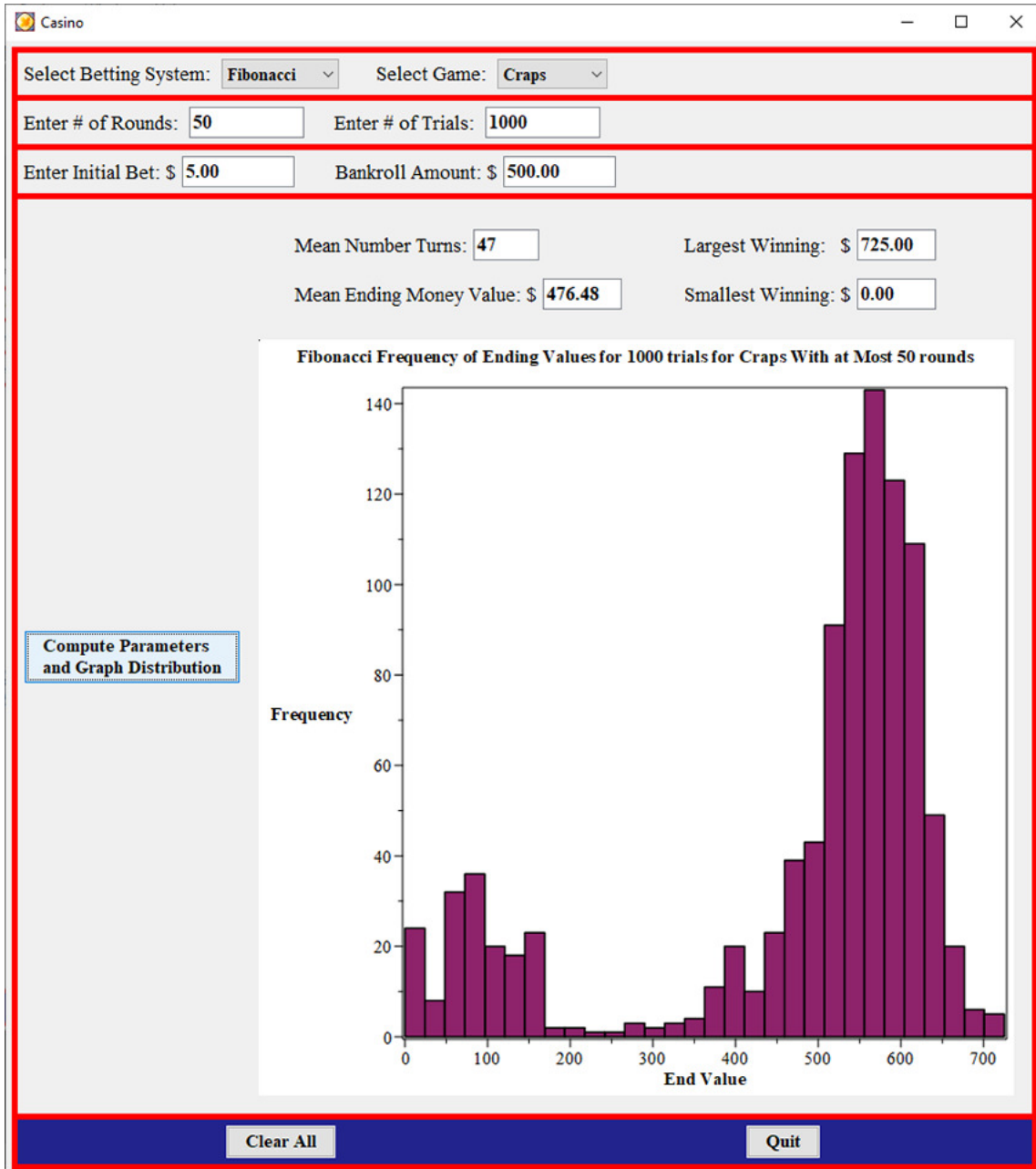Figure 7: Ending Money Values for Blackjack With at Most 50 Rounds Under Fibonacci System

Figure 8: Ending Money Values for Craps With at Most 50 Rounds Under Fibonacci System
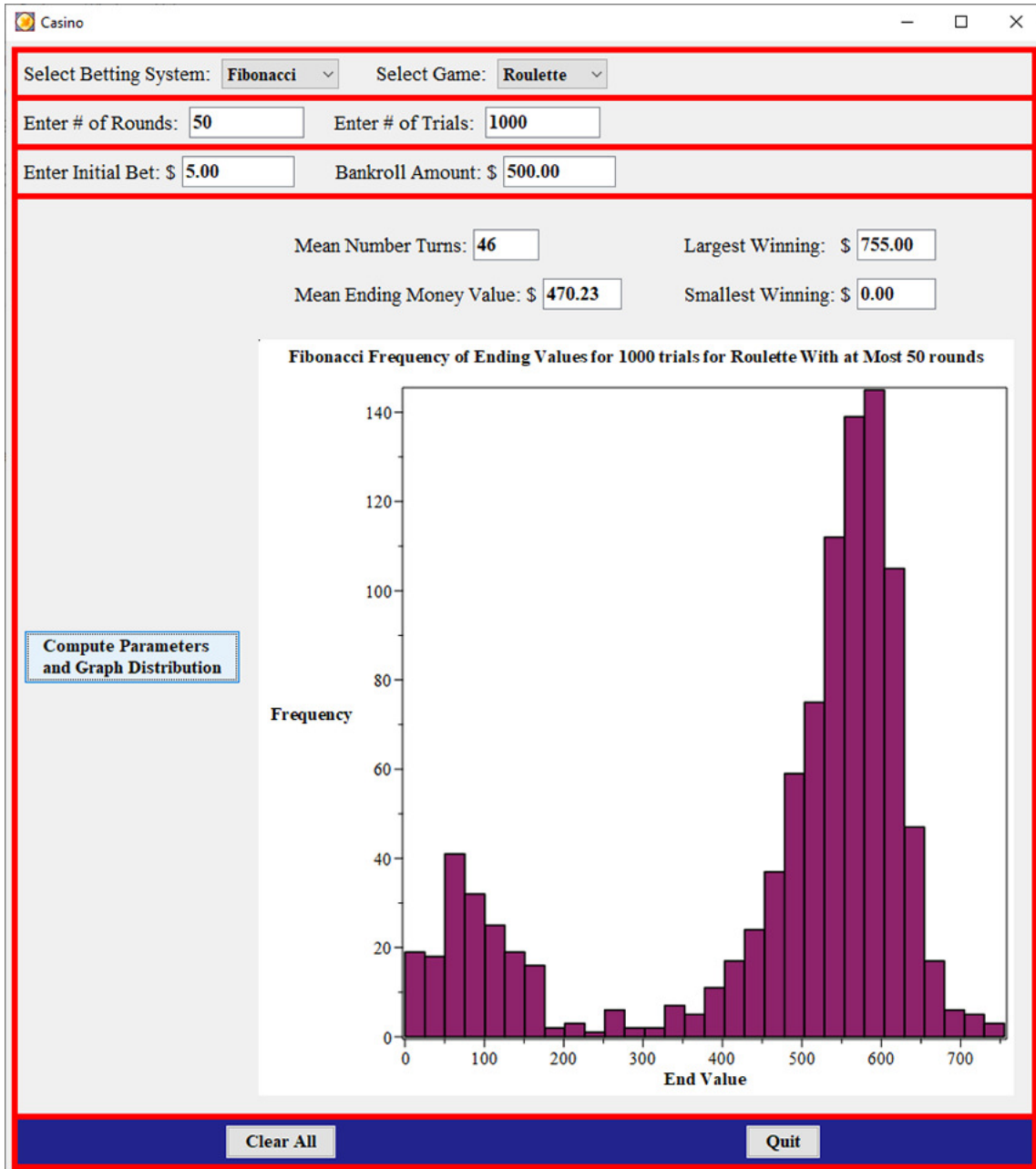
Figure 9: Ending Money Values for Roulette With at Most 50 Rounds Under Fibonacci System

4   Conclusions and Ideas for Future Work

As part of this project, simulations were also done using other number sequences such as Pi and Euler that show more randomness. These systems did not show promise from a gambling perspective though, as the key to a progressive betting system is the ability to regain losses as quickly as possible with a minimal number of wins.

For the analyses described in this paper, the Martingale betting system shows the most promise for potential profit in the short term, unless the probabilities of success on a given round are approximately 50%. However, under the Martingale system, if a player started by betting 1% of their beginning bankroll, then they would only be able to survive six consecutive losses before being unable to place the next bet. Even worse, if a player started by betting 5% of their beginning bankroll, then they would only be able to survive four consecutive losses before being unable to place the next bet. As such, the player would need to decide what it is really worth to them to have a profit potential. An intelligent strategy for the Martingale system would be for a player to identify how much profit they would need before leaving the game. For example, if a player had the goal of earning $25 profit in at most 25 rounds, then it would only take five wins when betting $5, and as soon as the player reached that goal, they could retire.

In contrast to the Martingale betting system, the Paroli betting system allows players to guarantee being able to play any specified number of rounds that they wish. More specifically, if the player chose to start by betting a certain percentage of their beginning bankroll, then they would be able to predict a minimum number of rounds they would be guaranteed of being able to play. For example, if a player started by betting 1% of their beginning bankroll, they would be guaranteed of being able to play a minimum of 100 rounds. Similarly, if they started by betting 5% of their beginning bankroll, they would be guaranteed of being able to play a minimum of 20 rounds. (This assumes the player completes 100 or 20 rounds, respectively, without three consecutive wins.) Another favorable property of the Paroli betting system is that players can double their bets until they earn two wins in a row, or four wins in a row, or any number of wins in a row they want to achieve. Recall though that regardless of however many consecutive wins a player wants to achieve in order to perceive the result as a success, the odds of reaching that number of wins would still need to be higher than the payoff. For example, two consecutive wins would result in a 3 to 1 payout, while the odds of winning two games in a row would be approximately 4 to 1.

Finally, recall that the Fibonacci betting system, like the Martingale betting system, allows bets to potentially become extremely high if players experience a large number of consecutive losses. So, similar to the Martingale system, with the Fibonacci system it would be better for players to start by identifying a profit amount as the goal rather than a number of rounds to be played. The fact though is that all betting systems have benefits and drawbacks, and so it ultimately comes down to the goals of the player. This project only aims to investigate systems with the end goal of allowing players to play for as long as possible and to reach a reasonable number of rounds with the hope of being entertained by a pleasant evening at a gambling table. The system that seems to best fit that criterion is Paroli, given that it is not for high-risk gamblers, and players can guarantee being able to play any specified number of rounds.

Further investigations could include developing an algorithm through which a player could choose a progressive betting system from among the three analyzed in this paper or possibly others, choose a casino game from among the three analyzed in this paper or possibly others, and then have the algorithm return to them the expected outcome of playing their chosen game under their chosen betting system. With the understanding that no system can guarantee a profit, the player could at least make better choices that lead to more pleasurable gambling experiences.

5   Supplementary Electronic Materials

[S1] Maplet that can be used to analyze the three betting systems and the three casino games considered in this paper:
https://www.appstate.edu/~klimare/casino.maplet.

References

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