The affordances of mathematical tasks in a learning management system

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Abstract

In this study, 400 randomly selected mathematics tasks in a widely used LMS in Finland designed to function as electronic learning platform for mathematics for primary and secondary school pupils were analysed using conceptual and procedural knowledge-based emphases. The analysis showed that approximately 29% of the selected 400 math problems emphasized conceptual knowledge. The results of this study suggest that the LMS consists of more math tasks emphasizing procedural knowledge than conceptual knowledge at primary and secondary school levels. These results are similar to the former findings of studies on Finnish mathematics textbooks. It can be concluded that mathematics tasks in the LMS promote the learning of procedural knowledge, but the criticisms regarding the materials and their versatility are still relevant. Understanding how conceptual and procedural knowledge is emphasized in different LMS helps current and future teachers to select and adapt teaching and learning materials and methods to better meet the needs of their students.

1. Introduction

A learning environment is often conceptualized as an environment where learning takes place. Typical dimensions of a learning environment include physical, social, technological, and pedagogical perspectives (e.g. [1], [2]). As teachers work in these learning environments, they must plan how to design environments to support students' learning as well as how to utilize the learning environments that they are provided. From this perspective, a theoretical framework known as The Activity-Centred Analysis and Design (ACAD) has been developed, which approaches the learning environments from the perspective of affordances (see e.g. [3], [4]). These various affordances are classified into three distinct categories: social, physical, and epistemic [4]. The idea in the framework is that teachers design the affordances for the learning environments before the teaching sessions, and during the session through active use of these different affordances the emerging learning environment is formed. Within this framework, social affordances refer to various social formations available in the classroom or online platform. Physical or set affordances include all the tools, artifacts, resources, and technologies. Epistemic affordances, for their part, are primarily focused on the tasks, sequences, and pace of tasks and content.

In this study, LMS is defined with reference to the research by Chaubey and Bhattacharya [9] as follows: LMS is a software application or web-based technology used to plan, implement, and evaluate a specific learning process [9]. The LMS platform of this study provides teachers with versatile tools for learning management and teaching [7]. It provides students and teachers opportunities for interactive and flexible mathematics learning, but it also helps students to improve their calculation skills and induce motivation to study mathematics [8], [10]. The LMS provides teachers with ready-made task repositories and learning paths that are already designed to suit, for example, a certain grade level. This LMS system is primarily used in the classroom or at home in addition to other study material. The LMS examined in this study considers the perspective of assessment by providing learning analytics data to both students and teachers about the completion of tasks, time use, and the correctness of answers. In this LMS system, teachers can also modify existing task repositories, create their own tasks, and choose the tasks they consider suitable for their students. For teachers to modify existing task repositories, they should consider what kind of epistemic affordances the tasks in the LMS system and repositories generally contain. The LMS of this study is widely used in Finland primary and secondary schools.

A key question that arises in this context is how to ensure the availability of high-quality mathematics teaching and learning materials in the digital learning environments and LMS because PISA results have suggested that transferring pencil-and-paper mathematical tasks as they are to electronic environments may decrease students' performance [11]. The goal of this study is to shed light on how conceptual and procedural knowledge is emphasized in mathematics tasks by primary and secondary school levels. Specifically, information is obtained about the type of content included in the tasks of the LMS system.

2. The need to assess conceptual and procedural knowledge of the mathematics tasks

The quality of mathematics tasks can be viewed from the perspective of mathematical content, which involves using theory to classify mathematical knowledge. A well-known theory in this regard separates procedural and conceptual aspects of mathematical knowledge [12], [13]. After an analysis of researchers' views, Haapasalo and Kadijevich [14] suggested the following characterization that fits viable theories of teaching and learning: Conceptual knowledge can be defined as an understanding of how concepts and problems form a network with each other. Haapasalo and Kadijevich [14] referred to the concepts in this semantic network as nodes and the connections between them as links. According to them, conceptual knowledge also includes the ability to navigate this network flexibly and to find concepts and their properties that are suitable for each situation or problem. Conceptual knowledge included the concepts, principles and relationships underlying the mathematical subject area and their understanding [12], [15]. More specifically, this included interpretation and construction of concepts and their attributes, procedures, functions, and perspectives [14]. Procedural knowledge can be defined as the ability to use and represent rules, algorithms, and procedures [12]. Haapasalo and Kadijevich [14] saw the foundation of procedural knowledge as processes formed by sequentially executed operations. Procedural knowledge referred to the dynamic and purposeful execution of rules, methods, or algorithms using certain presentation methods [14]. Therefore, procedural knowledge meant the ability to recognize the steps of a process and understand how the next step is carried out.

According to Haapasalo and Kadijevich [14], [16], [17] solving a math task utilization of conceptual knowledge typically required conscious thinking and knowledge of why the task is being solved, while procedural knowledge may consist of automated and subconscious steps. When solving a math task, the utilization of procedural information required the execution of automated calculation routines and knowledge of how to solve the task efficiently and accurately [12], [15], [16], [17]. In this case, there was a dynamic relationship between the two types of knowledge, and a clear boundary between them cannot be defined. Haapasalo [18] suggested that when solving a problem relying on procedural knowledge, two related rules can be successfully combined without understanding the underlying justifications. In general, conceptual, and procedural knowledge are strongly interconnected, and both are needed in solving many problems.

Research literature helps to group mathematics tasks into procedurally or conceptually focused areas [12], [15], [16], [19], [20] [21]. The complexity of tasks can be influenced by the number of procedures or concepts required to solve the task [21]. For example, Authors [22] presented a four-category classification tool that considers the complexity of tasks when evaluating procedural and conceptual knowledge within them. Haapasalo [20] also presented in his work a way to classify mathematics tasks utilizing conceptual and procedural knowledge. Additionally, Phuong [21] introduced a PCK taxonomy that can be utilized for classifying tasks and used in assessing students' mathematics learning.

Star [23] aimed to challenge the notion that procedural knowledge is shallow while conceptual knowledge is deep. For instance, she offered the concept of deep procedural knowledge, which included procedural flexibility that allows an individual to choose the most suitable method from various options in a problem-solving situation, such as solving an equation. On the other hand, knowledge of the relationships between concepts can be quite deficient and superficial, based on rote memorization rather than true comprehension.

When examining mathematics teaching practices, whether teachers should teach mathematics for procedural knowledge, conceptual knowledge, or a combination of the two is a question that arises [13]. This question can be examined through learning materials and mathematical tasks. The procedural–conceptual nature of mathematical tasks is revealed to each student individually based on the student’s existing knowledge and skills [24], [25], [26], [27], although mathematical task can be designed to be conceptually or procedurally oriented [15], [16], [18], [20], [21], [28]. From the point of view of primary school teachers’ professional relevance, mathematics textbooks are central because they are one of the most common epistemic affordances of mathematics in classroom [4] and the tasks given in textbooks shape learners’ perceptions of mathematical knowledge [29], [30].

The construction of connections between concepts can also be supported by other educational means, such as by using functional teaching and learning methods, increasing the use of illustration tools [31], or strengthening the role of interactions in learning mathematics [32]. In teaching arrangements, technology can create opportunities for dialogue about the emphasis on mathematical knowledge [33]. Many LMS systems enable the creation and introduction of conceptual tasks as part of the study of mathematics, making it possible to balance the dialogue around conceptual and procedural emphases in the mathematics learning process. According to Hurrell [34], mathematics teaching and learning materials should emphasize conceptual knowledge as this would allow learners to develop mathematical competence. In addition, mastering a calculation method without a conceptual understanding behind it does not produce permanent or very sustainable knowledge of the matter.

Research suggests that both conceptual and procedural knowledge are important for learning mathematics, and that the balance between the two are crucial for student achievement [7], [36]. A balanced approach to mathematics instruction should emphasize the development of both conceptual and procedural knowledge. Studies have suggested that students who have a strong conceptual knowledge of mathematics tend to perform better on math assessments and are better able to transfer their learning to new situations than students who lack this conceptual knowledge [36]. Procedural knowledge is also important for success on math assessments, especially for tasks that involve routine calculations or basic algorithms [15].

The effectiveness of mathematics tasks in promoting a balance between conceptual and procedural knowledge depends on several factors, such as the quality of tasks, teacher, and the background knowledge and motivation of the students. Well-designed mathematics tasks provide students with opportunities to develop both conceptual and procedural knowledge and encourage them to make connections between the two [37]. However, it should also be noted that other factors, such as how teachers use mathematics curriculum materials, may be more important in influencing student achievement than the balance of conceptual and procedural knowledge in mathematics learning materials [38], [39].

The findings of researchers on the biases in textbooks are causing serious concern in Finland. In Joutsenlahti's and Vainiopää's [19], [40] analysis studies, more than 80 percent of the tasks in Finnish elementary school mathematics textbooks turned out to be procedurally weighted. According to these researchers, the examination of mathematical concepts in textbooks was superficial and did not support the formation of connections between concepts. According to Viholainen et al. [41], Finnish secondary school mathematics textbooks emphasized mathematical procedures more than concepts and the connections between them.

Therefore, teachers should be made aware of contemporary research and literature on procedural and conceptual knowledge so that they can make informed decisions when selecting appropriate epistemic affordances of mathematics, such as the math tasks and the learning environments [45]. In addition, teachers' understanding about how the conceptual and procedural types of knowledge are emphasized in math tasks is essential when an LMS is used in the math classroom. This is especially true when the implemented technology offers the opportunities to choose whether to focused on the fluency of calculations and the execution of algorithms, which further contributed to develop students' understanding of mathematical concepts, or to focus on understanding concepts, which serves to develop calculations and the execution of algorithms [13], [18], [34].

3. Research Questions and Methods

This study examined epistemic affordances in a widely used LMS in Finland based on procedural and conceptual knowledge. An awareness of the procedural and conceptual knowledge involved in mathematical tasks can provide teachers with valuable information for choosing tasks when creating teaching and learning environments. As such, two research questions were formulated for this study:

Q1. What is the epistemic affordance of the LMS in primary school mathematics task examined from the perspective of conceptual and procedural knowledge?

Q2. What is the epistemic affordance of the LMS in secondary school mathematics task examined from the perspective of conceptual and procedural knowledge?

In this study, the epistemic affordance was formed from 400 electronic math tasks in the LMS which were randomly selected. The tasks were chosen based on a simple random sampling. In all, 220 of 881 tasks were selected from *Mathematics, Task Bank Primary School 2022–2023* in LMS, and 180 of 872 tasks were selected from *Mathematics, Task Bank Secondary School 2022–2023.* Both task banks are freely available to all teachers who have credentials to use the LMS and teachers can use the tasks as resources to develop their own courses in the LMS.

The analysis framework used in this study was built within the research group and has been used in two peer-reviewed articles [22], [42]. In this study, the analysis was conducted in three stages. In the first stage, the first author performed preliminary analysis by categorizing 400 mathematics tasks using the analysis framework [22], [42]. In the second stage, 120 of the 400 tasks (30%) were randomly selected for analysis by two other authors to enhance the inter-rater reliability of the categorization by Krippedorff's alpha values [43]. These 120 tasks were distributed into 66 elementary school tasks and 54 middle school tasks. Discrepancies in classification were resolved through consensus discussions to strengthen classification principles. Krippendorff's alpha values were calculated for the primary school ($\alpha = 0.83$) and secondary school ($\alpha = 0.90$) task classifications. Both Krippendorff's alpha values considered acceptable ($\alpha > 0.80$) [53]. In the third stage, the first author corrected the results of the preliminary analysis according to the established classification principles, and the analysis was completed. The results are reported in Tables 2 and 3. The tasks were analysed and classified into four categories in following way. Tasks requiring only symbolic representations and successive executions of the same algorithm or calculation to produce solutions fell into the categories of simple or complex procedural tasks. Task that can be solved with a one calculation and only require symbolic representations was classified the category of *simple procedural* tasks (see Table 1 for examples). When task solutions required more complex algorithms, or the execution of several algorithms or calculations was necessary, the task was classified into the category of the *complex procedural* task (see Table 1 for examples).

Tasks requiring connections between multiple representations, not only the symbolic form, fell into the category of simple or complex conceptual tasks. The task was classified into the category of *simple conceptual* tasks if it required identifying and ordering the different stages of the calculation. Typically, in the analysis, the solution required connections between multiple representations, not only symbolic form. (See Figure 5 and 6 for examples). Tasks were identified into the category of *complex conceptual* tasks when the concepts were presented in an open problem-solving format. The emphasis was on making decisions about the concepts or parts needed for the solution, as well as on organizing the connections between the parts and different types of representations to produce the solution (see Figure 7 and 8 for examples).

4. Results

This section first presents the basis for classifying tasks into four categories with the help of examples. After this, research questions 1 (Table 2) and 2 (Table 3) are examined using tabulation. Table 1 shows examples of procedurally weight tasks that emphasize procedural knowledge at Primary and Secondary school levels.

Table1: Examples of procedurally weighted tasks at Primary School and Secondary School

Procedural task types	Example
1. A Simple Procedural Mathematics Task at the Primary School Level	$\frac{3}{3} - \frac{1}{3} =$
2. A Simple Procedural Mathematics Task at the Secondary School Level	$-4 - (2) =$
3. A Complex Procedural Mathematics Task at the Primary School Level	$\begin{array}{r} 3{,}9 \\ -\ 1{,}1 \\ \hline \square \end{array}$
4. A Complex Procedural Mathematics Task at the Secondary School Level	Calculate, when $x = 7$. $10 \cdot x + 8$

Table 1 shows that the solution to tasks 1 and 2 is obtained by performing one subtraction operation. Therefore, these two tasks are examples of simple procedural tasks. For task 3 and 4, the solution is obtained by performing more than one calculation operation. Thus, these two tasks are examples of complex procedural tasks.

In each of the tasks below (Figures 1 and 2), generating a solution requires identifying and ordering the stages involved in the calculation. In Figure 1, subtraction calculation skills are not necessarily needed due to the pictorial representation provided. In Figure 2, equations and non-equations need to be recognized and classified. Thus, these two tasks are examples of simple conceptual tasks.

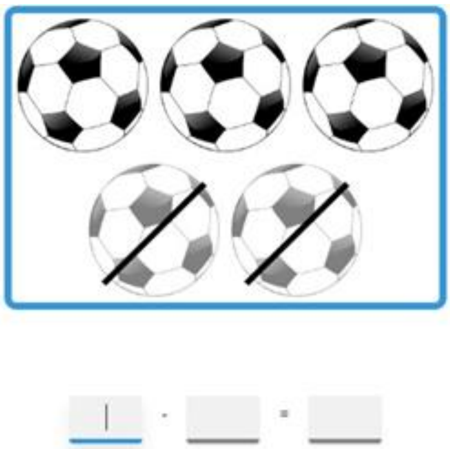

Figure 1: *A Simple Conceptual Mathematics Task at the Primary School Level*

3-(x+1)	x+x	x-(1-x)=5	2x+1
5x	x+13	x+5=10	5x=5

Equations	Non-equations

Figure 2: *A Simple Conceptual Mathematics Task at the Secondary School Level.*

The tasks below (Figures 3 and 4) are both presented in a problem-solving format, and decisions need to be made about the operations required to produce the solutions. Figure 3 requires identifying the rule behind the sequence and producing the solution. In Figure 4, a decision needs to be made about the order and kinds of calculations to be used to produce the solution. Thus, these two tasks represent complex conceptual tasks.

60	57				45	42

Figure 3: *Complex conceptual mathematics task (primary school)*

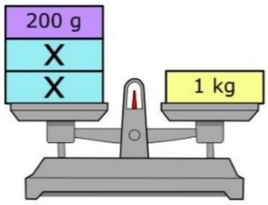

Figure 4. *Solve x. Complex conceptual mathematics task (secondary school)*

Based on the content analysis, 400 electronic mathematical tasks were classified into four categories. As shown in Table 2, 285 tasks emphasized procedural knowledge, and 115 emphasized conceptual knowledge. The greatest percent of tasks were procedurally complex mathematical tasks, and the least percentage of tasks were conceptually complex tasks. The number of tasks emphasizing procedural knowledge was two and a half times the number of tasks emphasizing conceptual knowledge. Interestingly, there were more complex procedural tasks than simple procedural tasks. In contrast, there were almost four times as many simple conceptual tasks compared to complex conceptual tasks.

Table 2. 400 Math Tasks Classified Based on Conceptual and Procedural Knowledge and Difficulty (Simple or Complex)

Task type	Simple	Complex	Total
Procedural	123 (30 %)	162 (41 %)	285 (71 %)
Conceptual	91 (23 %)	24 (6 %)	115 (29 %)

Table 3 shows how the 400 mathematical tasks were classified into four categories based on school level. In comparison to examining the set as a whole, there are several differences. For example, the number of procedural tasks in the primary school is almost triple that of the conceptual tasks. However, that same comparison is only double at the secondary school. Additionally, there are over 8 times as many simple conceptual tasks at the primary level, while a little over twice as many at the secondary level.

Table 3. 400 Math Tasks Classified Based on Conceptual and Procedural Knowledge and Difficulty by School Level (Primary and Secondary)

Task type	Primary school	Secondary school
Simple procedural tasks	76 (35 %)	47 (26 %)
Complex procedural tasks	88 (40 %)	74 (41 %)
Simple conceptual tasks	50 (22 %)	41 (23 %)
Complex conceptual tasks	6 (3 %)	18 (10 %)

5. Conclusions and Discussion

The focus of this study was on analyzing freely available mathematics tasks in the LMS as epistemic affordance of primary and secondary school mathematics based on whether the task emphasized conceptual or procedural knowledge and the difficulty of the tasks. The results of this study showed that the tasks as an epistemic affordance consist of more math tasks emphasizing procedural knowledge than conceptual knowledge at both school levels. The most tasks were categorized to the category of complex procedural tasks at both school levels. It can be noted that when comparing primary school tasks to secondary school tasks, the number of simple procedural tasks decreased, and the number of complex conceptual tasks increased. However, the percentage of complex procedural and simple conceptual tasks were comparable between the two school levels.

This study has some factors related to its reliability and generalizability, which should be considered when interpreting the results and conclusions. First, the research is repeatable using the analysis framework and task banks since the tasks are easily accessible. Second, the randomly selected task sample was large enough (approx. 25% of all tasks), but this must be considered when generalizing the research results. The findings of this study cannot be generalized to all materials obtained from the LMS. It is possible that some other materials in the LMS, such as grade-specific mathematics study paths, contain tasks that emphasize conceptual knowledge more than the materials selected for this study do. Third, the analysis framework used in this study is scientifically reported. In addition, the reliability of this study and classification was confirmed by calculating Krippendorff's alpha values, but it was only done for 120 tasks. In addition, the discriminating ability of the analysis framework and the classification model can be considered good, but placing the task in one of the four categories requires the researcher's own interpretations and decision-making, which requires good theoretical knowledge. Thus, caution should be exercised when generalizing the results of this study.

The results that there are more procedurally weighted tasks than conceptually weighted tasks are nearly similar to the findings results of Finnish mathematics textbooks [19], [40], [41]. Based on the results (Table2), 71 percent of tasks still seemed to focus on procedural knowledge, and complex procedural tasks were the most common tasks at both school levels. It can be concluded that mathematics tasks in the LMS emphasized the learning of procedural knowledge [7], [8], [10]. For this reason, criticisms regarding the materials and their versatility, which emerged based on earlier research, are still relevant. When mathematics tasks from textbooks are transferred to the LMS without adaptation, their conceptual–procedural emphasis does not change, only the format changes. In such a case, the possibilities of the LMS as an epistemic affordance to promote the exploration and manipulation of mathematical concepts to achieve conceptual knowledge will be forgotten [5], [7].

Results reveal that mathematics tasks in LMS also promote the learning of conceptual knowledge at both school levels and this trend can be considered good. Based on the distribution of tasks, the situation compared to the mathematics textbooks seems to have improved in that the primary school LMS and the secondary school LMS appear to integrate slightly more conceptual tasks, but the primary school LMS more closely aligns with the previous textbook studies [19], [40], [41]. The lower number of conceptual tasks in the sample may explain that not all tasks are suitable for automatic checking, and it is easier and faster to perform repetitive tasks aimed at fluency in counting.

From the perspective of teachers and teacher education, this research provides information about each LMS and its potential in the planning, implementation, and evaluation of mathematical tasks. Recommendations for the use of various electronic learning environments and the LMS come from the Finnish National Core Curriculum for Basic Education 2014, which underlines the significant role of ICT in mathematics across all grade levels [44]. For this reason, a variety of technologies are increasingly being used in mathematics classrooms. It is possible to balance epistemic affordance in the LMS by analysing and reviewing the conceptual and procedural emphases of mathematical tasks with the framework used in this study [22], [42].

Differentiating conceptual and procedural knowledge from each other is both a theoretically and methodologically demanding task [14]. The teacher’s knowledge, skills, and abilities in selecting and assigning mathematics tasks in the LMS, utilizing both conceptual and procedural knowledge, become necessary [34]. For teachers to effectively select tasks based on knowledge emphasis, it is crucial for them to recognize how conceptual and procedural knowledge are emphasized in various mathematics tasks [33], [34]. Understanding how conceptual and procedural knowledge is emphasized in different LMS tasks will help current and future teachers select and adapt teaching materials and methods to meet the needs of their students.

It is notable that there are not any components in the two LMS and task repositories to help teachers meet their classroom needs of procedural and conceptual knowledge, but the LMS is flexible and facilitates quick material updates. For this reason, its potential to promote learning of conceptual knowledge can be viewed from two perspectives of epistemic affordance. Firstly, the epistemic affordance of tasks emphasizing conceptual knowledge in the LMS can be increased. Secondly, the existing epistemic affordance of the LMS can be improved by adding information about the classification of the conceptual and procedural knowledge of the tasks.

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