

Problem Corner  
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Thinking Skills and Technology

**Question 1** Solve the inequality  $f(x, y, z) = 2x - 3y - 2z - 11 + 5\sqrt{x^2 + y^2 + 11 + 4z^2} \leq 0$ . Use CAS, if it is convenient.

**Question 2** Let  $f(x, y, a) = 1 - \sqrt{x^2 - 2ax + y^2 - 4y + 5}$ . The expression gets its maximum value only for a single pair  $(x, y)$ . Find the range of the parameter  $a \in [-1, 1]$ .

**Question 3** Let  $N$  be the Nagel point of triangle  $ABC$ . Let  $T_A$ ,  $T_B$ , and  $T_C$  be the extouch points at which the  $A$ -excircle meets line  $BC$ , the  $B$ -excircle meets line  $CA$ , and  $C$ -excircle meets line  $AB$ , respectively. Let  $P_A$  be a point on  $AT_A$  such that  $AP_A = NT_A$ . Let  $P_B$  be a point on  $BT_B$  such that  $BP_B = NT_B$ . Let  $P_C$  be a point on  $CT_C$  such that  $CP_C = NT_C$ . Find the incenter of  $P_AP_BP_C$  triangle.

**Note:** "Nagel point" from Wikipedia:

In geometry, the Nagel point is a point associated with any triangle. Given a triangle  $ABC$ , let  $T_A$ ,  $T_B$ , and  $T_C$  be the extouch points in which the  $A$ -excircle meets line  $BC$ , the  $B$ -excircle meets line  $CA$ , and  $C$ -excircle meets line  $AB$ , respectively. The lines  $AT_A$ ,  $BT_B$ ,  $CT_C$  concur in the Nagel point  $N$  of triangle  $ABC$ . The Nagel point is named after Christian Heinrich von Nagel, a nineteenth century German mathematician, who wrote about it in 1836.