# PROBLEM CORNER

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1. In a remote, semi-barbaric kingdom[1], the need for a strong army led the King to promulgate a cruel law: To raise the number of young men available, no family should have more than one daughter. Thus, every woman in the kingdom had a certain number of children, and the last of these (and *only* the last) was a girl. What was the final proportion of boys and girls? (Of course, you should assume that the events "having a child" are independent from each other, with a probability $p = 1/2$ for a boy and $p = 1/2$ for a girl).

2. Surely the reader has heard about "Flatland", by E. Abbott, a book that describes the life of geometric creatures in the Euclidean plane. It is a somewhat boring world; for example, you can consider the fact that the only permitted motion in it, is along straight lines. Here is the proof: Given any smooth path $x : I \subset \mathbb{R} \to \mathbb{R}^2 - \{0\}$, let us see that there exists a unit vector $u \in \mathbb{R}^2$ such that $x(t) = |x(t)|u$. To this end, notice that because $x(t)$ does not take the value 0, we can compute (denoting by a point the derivative with respect to $t$)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{x(t)}{|x(t)|}\right) = \frac{|x|^2\dot{x} - \langle x, \dot{x}\rangle\, x}{|x|^3}\,,$$

where $\langle\cdot,\cdot\rangle$ denotes the scalar product in $\mathbb{R}^2$, and $|\cdot|$ the Euclidean norm. Of course, $\mathbb{R}^2 \subset \mathbb{R}^3$, so we can make use of the identity

$$(u \times v) \times w = \langle u, w\rangle\, v - \langle v, w\rangle\, u\,,$$

which is valid for any vectors $u, v, w \in \mathbb{R}^3$. Thus,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{x(t)}{|x(t)|}\right) =& \frac{1}{|x|^3}\,((x \times \dot{x}) \times x)\\ =& -\frac{1}{|x|^3}\,((\dot{x} \times x) \times x)\\ =& -\frac{1}{|x|^3}\,(\dot{x} \times (x \times x))\\ =& -\frac{1}{|x|^3}\,(\dot{x} \times 0) = 0\,,\end{aligned}$$

and there must be a constant vector $u \in \mathbb{R}^2$ such that

$$\frac{x(t)}{|x(t)|} = u\,.$$

Is life in the plane really so boring?.

[1] Yes, this is a little homage to Stockton's "The Lady or the Tiger".