PROBLEM CORNER

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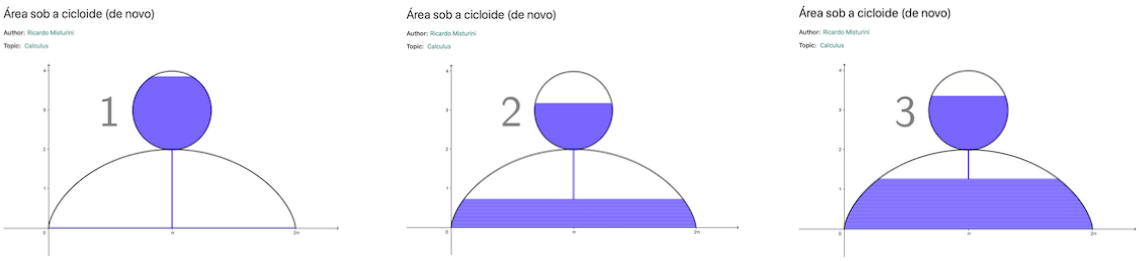
MOTIVATION

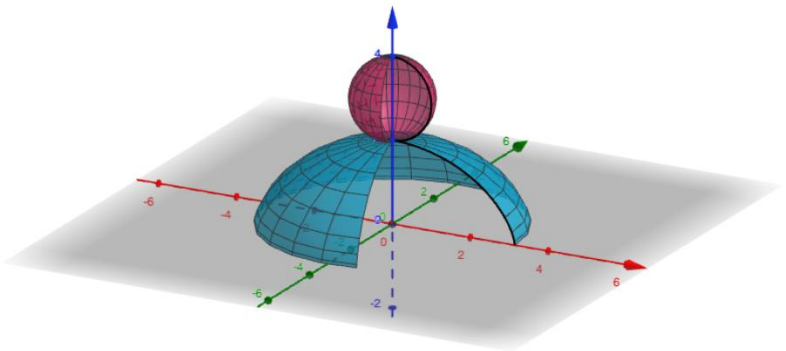
Cycloid is a well-known curve generated by the trace of a point on a circle, which rolls along a straight line without slipping or gaps. Historically it was already studied by several mathematicians and based on such contributions, it is nowadays a popular and important curve due to its physical interpretations. It is also quite common in Calculus courses, as a typical Integration exercise, to show that the area under one arc of this curve is equal to 3 times the area of the correspondent generator circle. A nice applet on GeoGebra Platform to illustrate (not proving) this fact is shared bellow:



https://www.geogebra.org/m/hacg6ex6

PROBLEM 1

Is the same relation still valid when we consider a 3D extended version of this? That means, does this relation remain for the volume under the revolution of the cycloid around its axis of symmetry and the volume of the corresponding sphere[1]?



HINT: For those who are not familiar with Calculus, it might help to consider the revolution of a right triangle, which has its shorter sides equal to the height of the cycloid and half the period of the cycloid (2 and π, respectively, when we consider the radius of the original circle equal to 1). The GeoGebra applet in the link below may be useful.

https://www.geogebra.org/m/tykauqhp

PROBLEM 2

Find (and prove) the ratio between the volume of the solid generated by the rotation of the cycloid around its axis of symmetry and the volume of the corresponding sphere.

[1] Sphere generated by a complete revolution of half of the circle which originally generated the cycloid.