

# PROBLEM CORNER

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## PROBLEM 1

The book (Lam & Pope) attributes the following pentagon construction from an A4 paper to David Collier, which it notes gives an approximately regular pentagon (Figure 1).

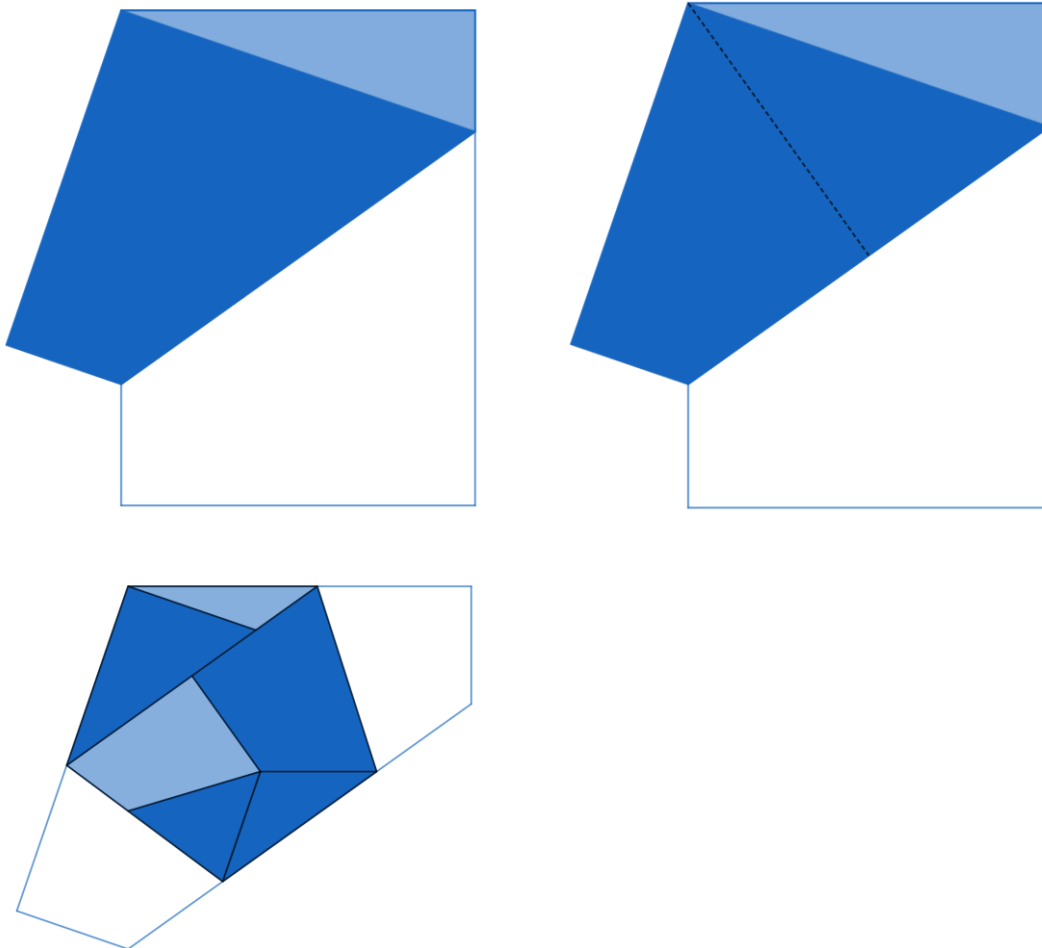


Figure 1. Pentagon construction

What should be the ratio of the sides of the rectangle so that the resulting pentagon is a regular pentagon?

## Reference

Lam, T. K., & Pope, S. *Learning Mathematics with Origami*. Association of Teachers of Mathematics.

## PROBLEM 2

### MOTIVATION

There is a famous problem, sometimes referred to as the “water well problem” (Goddijn & Reuter, 1995; Büchter & Leuders, 2005, p. 33), dates back to Descartes and Dirichlet who stated the problem in 1644 and 1850, respectively: “How can a plane be divided into areas (polygons) so that each point in an area is closer to the generating point than to any other generating point?” (Fisher, 2004)

This problem has been used in didactical settings to introduce the idea of perpendicular bisectors in problem-oriented teaching (e.g., Holzäpfel et al., 2016; Möller & Rott, 2017).

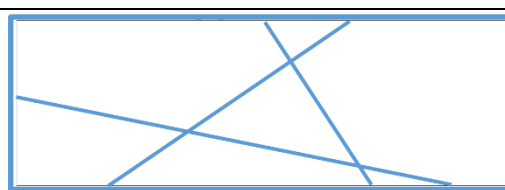
### PROBLEM AND VARIATIONS

Here, we want to pose a similar yet quite different problem, to the water well problem described above. You still want to reach water as quickly as possible:

#### The canal problem

The map shows a parcel of land. There are three canals in this area.

Develop a partition of the areas into regions in a way that from each place in a region the canal in that region is the closest.



The canal problem was derived from the water well problem as a variation, by slightly altering the conditions of the problem (the distance to the lines instead of points in this case) (cf. Brown & Walter, 1983; Silver, 1994). Can you pose an easier or a more difficult problem by varying the canal problem? Does the solution strategy change when there are fewer or more canals? What if there were (curved) rivers instead of (straight) canals? What if there were lakes (that do or do not look like circles)?

### References

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- Fisher, J. (2004). Visualizing the Connection Among Convex Hull, Voronoi Diagram and Delaunay Triangulation. Unpublished paper. <https://pages.mtu.edu/~shene/PUBLICATIONS/2004/Hull2VD.pdf>
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- Holzäpfel, L., Rott, B., & Dreher, U. (2016). Exploring perpendicular bisectors: The water well problem. In A. Kuzle, B. Rott, & T. Hodnik Čadež (Eds.), *Problem Solving in the Mathematics Classroom – Perspectives and Practices from Different Countries* (pp. 117–130). Münster: WTM.
- Möller, A. & Rott, B. (2017). Können durch problemorientierten Unterricht in derselben Unterrichtszeit vergleichbare Schülerleistungen erzielt werden? In U. Kortenkamp & A. Kuzle (Eds.), *Beiträge zum Mathematikunterricht 2017* (pp. 673–676). Münster: WTM.
- Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics*, 14(1), 19–28.