PROBLEM CORNER

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Introduction

In the ATCM 2024 Conference, we contributed with an article (see [1]) concerning the study of the following geometric question: *Consider a triangle ABC. Study the geometric locus of points P such that $\angle PBA = \angle ACP$.*

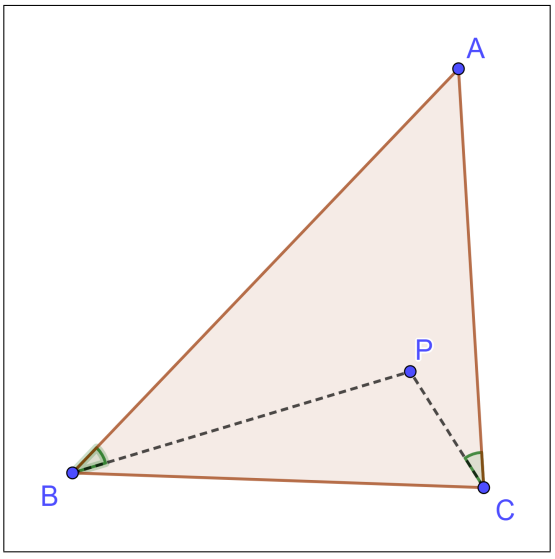

Figure 1: When $\angle PBA = \angle ACP$?

While working on this topic, which we carried out with the help of the `GeoGebra` mathematical software, some misprints while typing the commands to get our results led us to variations of our initial problem (the moral is that even silly mistakes can lead to new discoveries ;)), which we propose here now for further investigation.

The questions below are somehow open in nature, and we invite the readers to offer the best answers they can get. Exploration with mathematical software such as `GeoGebra Discovery` (see [2]) will probably help to get interesting insights into them.

Problem 1 *Consider a triangle ABC. Find the geometric locus of points P such that $\angle PBA = \angle PCB$ and study its properties.*

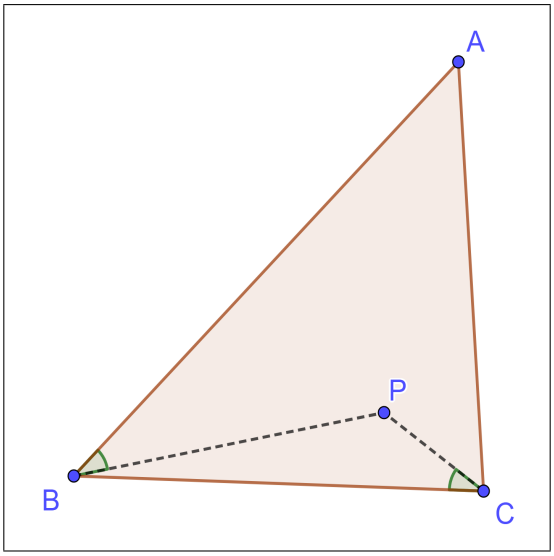

Figure 2: When $\angle PBA = \angle PCB$?

Problem 2 *Consider a triangle ABC. Find the geometric locus of points P such that $\angle APB = \angle CPA$ and study its properties.*

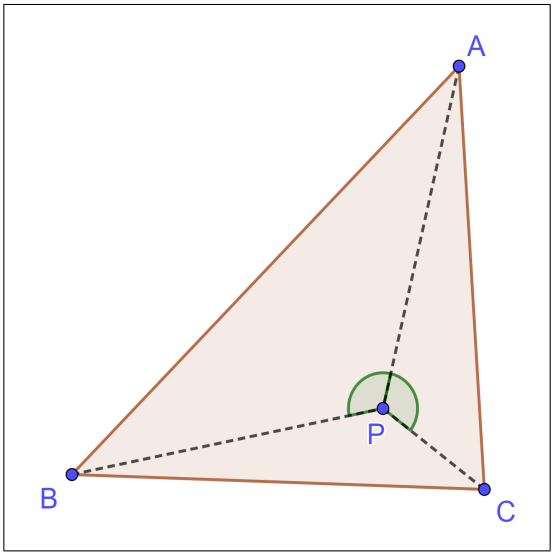

Figure 3: When $\angle APB = \angle CPA$?

References

[1] Recio, T., Ueno, C. (2024). Automated reasoning tools for dealing with elementary but intriguing geometric loci. In: Wei-Chi Yang, Douglas Meade, Weng Kin Ho (eds), Proceedings of the 29th Asian Technology Conference in Mathematics, (ATCM 2024). pp. 282-291. Published by Mathematics and Technology, LLC (`http://mathandtech.org/`), ISSN 1940-4204 (online version).

[2] GeoGebra Discovery. Available online (accessed on 19 November 2024): `https://github.com/kovzol/geogebra-discovery`