

**Question 1:** Are the following two curves the same?

Given a circle  $O$  centered at point  $O$  with a radius 5 (we will name all the circles by the center points from now on),  $A$  is a fixed point on circle  $O$  and  $B$  is the point moving around on circle  $O$ . For each such point  $B$ , draw a circle centered at  $B$  with a radius  $|BA|$ . Curve 1 is the boundary of the green region generated by all the circles centered at  $B$  as shown in Figure 1.

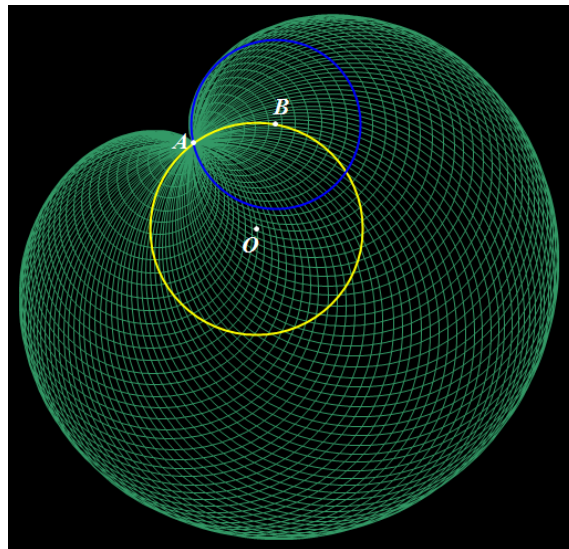


Figure 1

Circles  $O$  and  $P$  are both of radius 5 and tangent to each other at point  $A$  (the sharp edge). Being tangent to circle  $O$ , circle  $P$  rolls around circle  $O$  as illustrated in Figure 2. Curve 2 is the path of point  $A$ 's movement.

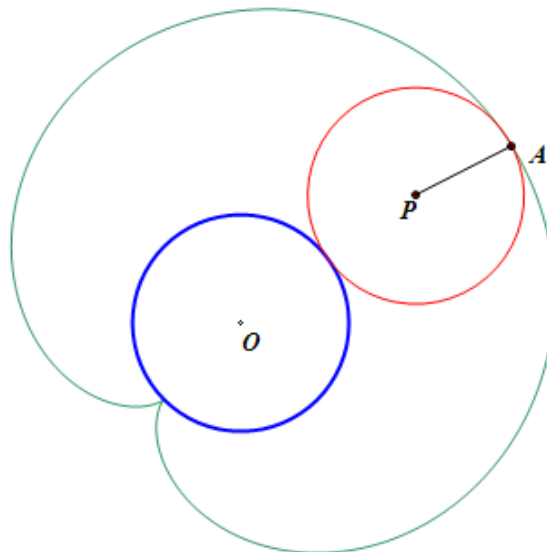


Figure 2

Is curve 1 identical to curve 2? Prove your answer.

**Question 2:** Are the following three curves the same?

Curve 3 is the edge of the purple region obtained by moving a line segment of length 5 along the x and y axes as shown in Figure 3.

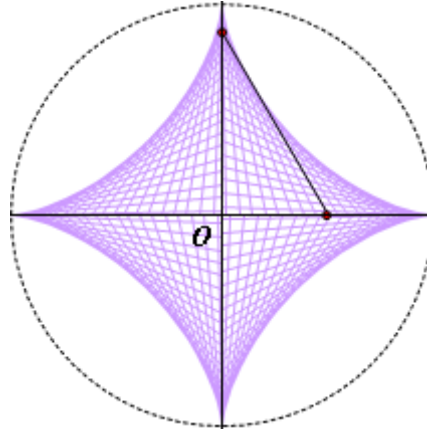


Figure 3

Curve 4 is the boundary of the orange region generated by all the ellipses  $E$  centered at the origin for which the sum of the lengths of the semi-major and semi-minor axes is always 5 as in Figure 4.

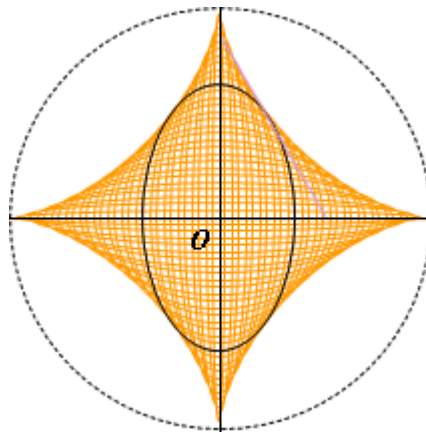


Figure 4

Given a circle  $O$  with a radius 5,  $P$  is another circle of radius 1.25 inside and tangent to circle  $O$  at point  $A$  (one of the sharp corners). Curve 5 is the trace of the movement of point  $A$  when  $P$  rolls around inside circle  $O$  as demonstrated in Figure 5.

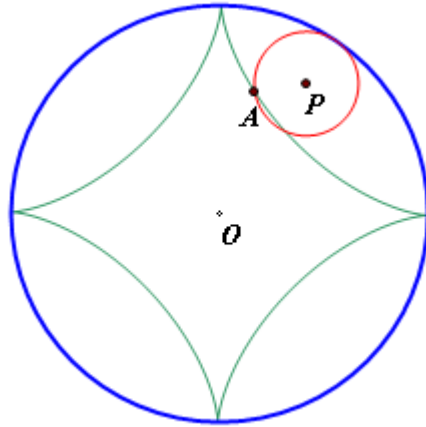


Figure 5

Do curves 3, 4 and 5 represent the same curve? Prove your result.