

PROBLEM CORNER

Provided by Jen-chung Chuan

E-mail: jcchuan@gmail.com

Problem 1

Let the sequence of integers $\{a(r,s) : s = 1, 2, \dots, 2^{r-1}, r = 1, 2, \dots\}$ be given by $a(1,1) = 1$,

$a(r+1,s) = a(r, (s+1)/2) + 1$ if s is odd, else $a(r+1,s) = a(r+1,s-1) * a(r, s/2) + 1$.

Show that the sum of reciprocals

$1/a(r,1) + 1/a(r,2) + \dots + 1/a(r, 2^{r-1})$ converges to $\pi/4$ as r approaches to infinity.

Problem 2

Construct 24 circles each touching exactly four others. (In space or on a plane, it doesn't matter.)

Problem 3

Construct five points forming the vertices of a regular pentagon using compass only.