PROBLEM CORNER

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Problem 1

A student uses a simple theodolite to measure the angle of elevation above the horizontal of a distant radio tower. The resulting angle is θ_1=1°25'29.907" above horizontal. He then moves exactly *d*=50 meters closer to the tower and measures the angle again, giving θ_2=1°25'55.546". What is the height of the tower *h* in meters, measured to the nearest centimeter? Be sure to include the curvature of the earth in your calculations, if necessary, with R_e=6371km. Assume that the measurements are made from a height of *m*=2 meters above the ground. Figure 1 illustrates these measurements. Note: the figure is not to scale.

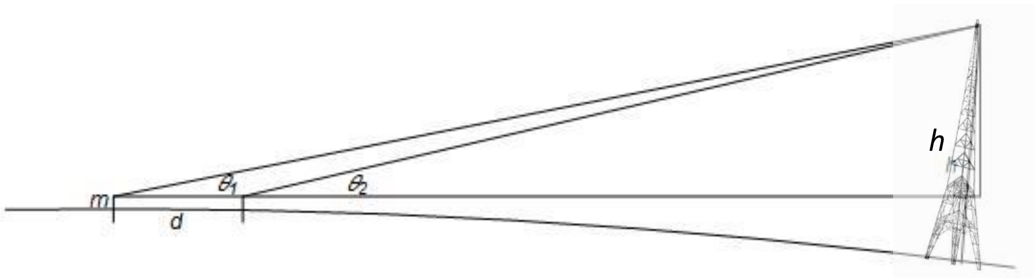

Figure 1 - Tower Measurements

Problem 2

A trebuchet, as illustrated in Figure 2, is a medieval machine for launching stone projectiles against enemy castles. The path of such a projectile (as shown in Figure 3) is modeled by a parabolic path with equation: $y = a\,x^2 + b\,x + c$. A set of two dimensional (x, y) measurements are made and recorded in the table below, where, unfortunately, the x value for y = 631 is missing.

X	5	8	10	13	xxxx
y	576	697	740	770	631

These points do not lie exactly on a parabola, so a linear least squares method was used to fit a parabola to the points by solving the matrix equation $A^T A\,x = A^T b$ where the matrix A and the vector b represent the set of linear equations created by plugging the (x, y) coordinates of the points into the parabola equation $a\,x^2 + b\,x + c = y$. The exact result of the least squares fit was the equation:

$$y = \frac{7839578}{2592212}x^2 + \frac{1357123223}{1728142}x + \frac{2692232981}{10368852}$$

A plot of the fitted equation along the four known points is shown in Figure 3. Your challenge is to determine exact value of the missing x coordinate of the 5th point that will give this least squares solution.

Figure 2 - A Projectile Launcher (Trebuchet)

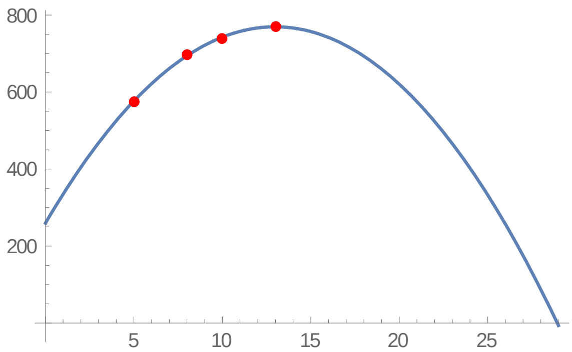

Figure 3 – The Projectile Path