

## PROBLEM CORNER

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### Problem 1

A series is just an infinite sum. It can be defined as the limit when  $n$  goes to infinity of the  $n$ th partial sums (that is, the sum of the first  $n$  terms). In this problem we consider the sum of a series in which the coefficients satisfy a linear recurrence. Compute the sum of the series

$$\sum_{n=0}^{\infty} a_n x^n,$$

where the sequence  $(a_n)$  satisfy  $a_0 = 1$ ,  $a_1 = 2$  and

$$5a_n + 2a_{n-1} - 4a_{n-2} = 0$$

### Problem 2

Infinite products, first studied by Euler and developed extensively by Weierstrass, can be defined analogously to series, as the limit of their partial products. Compute the infinite product

$$\prod_{n=0}^{\infty} \frac{a^{2^n} + 1}{a^{2^n}}$$

where  $a > 1$

**Remark:** While these problems can be done “by hand” (none of them requires maths beyond elementary calculus of limits), it is much more interesting to do some experimentation first with a CAS (such as Maxima) to get an idea of the relations and patterns between the inputs of the problems, which can lead to educated guesses for the solutions.