PROBLEM CORNER

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Problem 1

Consider the experiment of rolling a six-sided fair die. The aim of this problem is to illustrate the law of large numbers in identifying the true mean, μ, of the distribution when a die is rolled once. To accomplish this, do the following.

(a) Roll the die 5 times and calculate the sample mean of the observations. For example, the sample mean for the observations {5, 3, 4, 6, 1} is $\overline{x} = 3.8$. Repeat this with 10, 30, 50, 100, and 200 trials. Plot the sample mean $\overline{x}$ (vertical axis) against the number of trials (horizontal axis). What does $\overline{x}$ converge to? By the law of large numbers, the sample mean should gradually approach the true mean as the number of trials increases.

(b) Repeat the entire process in part-(a) using a software with 1, 2, 3, ..., 1000 trials. This should provide a better illustration of the law of large numbers. Find an approximate value of the true mean.

(c) Calculate the exact value of the true mean. Use an intuitive approach or use the knowledge taught in elementary statistics courses. Provide a rationale for your answer.

Problem 2

Consider the experiment of rolling an N-sided fair die, where the number of sides N is unknown. When the die is rolled, the minimum possible value is 1 and the maximum possible value is N. Suppose, one observes the following data when the die is rolled 10 times. Find a reasonable estimate for N.

Observed data									
10	14	4	3	17	15	6	19	2	9

A widely used approach to solve this type of problem is the maximum likelihood estimation that involves forming the likelihood function $L(N)$ and maximizing it with respect to N. To do so, one easily identifies that the trials are independent and each of the 10 observations has the identical probability of $1/N$ to be observed. Thus, the probability that the observations are obtained jointly is

$$\begin{aligned} L(N) &= \frac{1}{N} \times \frac{1}{N} \times ... \times \frac{1}{N} \\ &= \left(\frac{1}{N}\right)^{10}. \end{aligned}$$

The N that maximizes $L(N)$ is called the maximum likelihood estimate for N. Therefore, finding a reasonable estimate for N involves completing the following steps: (i) find the N that maximizes $L(N)$, which is the estimate in general, (ii) apply this to the given observed data for calculating the estimate for N.