PROBLEM CORNER

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In view of problems posted at *[4]*, we post the corresponding problems in 3D as follows.

Example 1 *We are given two concentric spheres centered at* $O = (0, 0, 0)$ *of radii of* a *and* b *(with* $a < b$*) respectively. See Figure 1, that is generated by GInMA [1] below. The unit sphere is depicted in blue and the sphere of radius* 2 *is the one in yellow. We are given a moving point* A *on the unit sphere and extend the ray* OA *to intersect the outer sphere at a point* B*. Next, we project point* B *onto the plane* E *(in purple), which is a plane that passes through* A *and is parallel to the* xy *plane. Denote by* P *the projection of point* B *in* E*. (In other words, the vector* AP *is perpendicular to the normal vector of the plane* E*.) Find the locus for the point* P *(See Figure 1).*

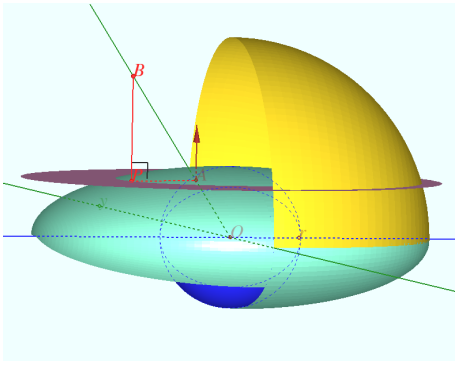

Figure 1. Generating an ellipsoid from two concentric spheres.

In order to generalize the idea of obtaining a locus through perpendicular projections, we consider the following cardioid surface below:

Example 2 *We are given a sphere centered at* $O = (0, 0)$ *with radius of* r_0*, and the cardioid surface* S*, by rotating* $[x(t), y(t)] = [a(1 - \cos t)\cos t + a, a(1 - \cos t)\sin t]$*, where* $t \in [0, 2\pi]$*, around the* $x-$*axis. Let* A *be a moving point on the sphere and we extend the ray* OA *to intersect the outer cardioid surface at a point* B*. Next, we project point* B *onto the plane* E*, which is a plane that passes through* A *and is parallel to the* xy *plane. Denote by* P *the*

projection of point B in E. In other words, the vector AP is perpendicular to the normal vector of the plane E. Find the locus for the point P . (See Figures 2(a)-2(c))

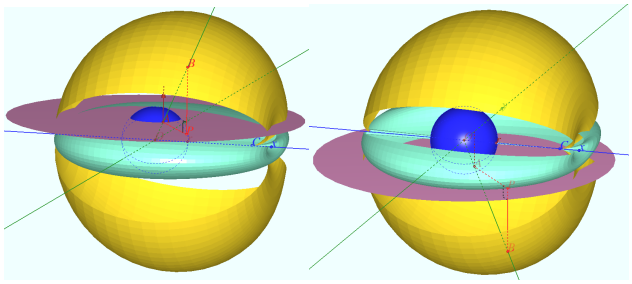

Figure 2(a) A sphere, cardioidal surface and locus

Figure 2(b) Locus surface when the point A varies

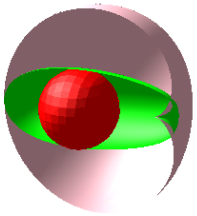

Figure 2(c) Locus generated by MAPLE

References

[1] Geometry in Mathematical Arts (GInMA): *A Dynamic Geometry System*, see http://deoma-cmd.ru/en/Products/Geometry/GInMA.aspx.

[2] Geometry Expression, see http://www.geometryexpressions.com/.

[3] Maple: A product of Maplesoft, see http://maplesoft.com/.

[4] Problem Corner from the Electronic Journal of Mathematics and Technology, February 2019: https://php.radford.edu/~ejmt/ProblemCornerDocs/eJMT_ProblemCorner_Problems_Feb201

[5] Yang, W.-C. See Graphs. Find Equations. Myth or Reality? (pp. page 25-38). Proceedings of the 20th ATCM, the electronic copy can be found at this URL: http://atcm.mathandtech.org/EP2015/invited/2.pdf, ISBN:978-0-9821164-9-4 (hard copy), ISSN 1940-4204 (online version), Mathematics and Technology LLC.