Math Games
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Computer games may not only be entertainment, but also assist in the development of logic and analytical thinking. **Math games** help people learn mathematics in a fascinating way, and more deeply understand various areas such as Mathematical Statistics, Finance, Game Theory, Probability Theory, etc.

The following problems are created in form of games. The first problem doesn't require specific knowledge of mathematics. All we need is logical reasoning capabilities. The next two problems are **economic games**. We may just play and try to win the games intuitively. However, in problems 2 and 3 which are both economic games, we can obtain a so–called **optimal strategy** to maximize the player's benefit on average. The **optimal strategy** may be found by treating the game as solving a problem. To find correct solutions we need to be familiar with **Game Theory** and **Probability Theory**, understand concepts such as a **probability** of an event and its measurement, mathematical **expectation**, the mean value of a random variable, payoff calculation, optimal strategy, etc.

Problem 1

In Koschei's kingdom, there are four water sources numbered 1, 2, 3, and 4, whose taste and appearance are not different from the popular Russian Vodka. But Koschei's spring water has the following feature: it acts as a poison and a person has to take antidote to avoid death in 5 minutes after drinking a glass of Koschei's spring water. Koschei's water can be drinked only in whole glasses. If one drank two or more glasses of Koschei's water successively from a given spring, or one glass of water and one or more glasses of Vodka, the result is the same as he only drank one glass of Koschei's water. Suppose one drank one glass of Koschei's water from either springs 1, 2, or, 3. To obtain an antidote, one would then have to drink a glass of Koschei's water numbered higher than the first glass. If one drinks a glass numbered lower than the first glass, then the person would die instantly. Everyone has access to water from the springs numbered 1, 2 and 3. However, only Kischei can take water from the spring 4. Therefore, only Kochei can obtain an antidote when drinking a glass of water from spring 3.

Gamer Ivan challenged Koschei to a duel. Under their agreement, Ivan and Koschei sit in their shelters before the duel. They have access to water (except water from spring 4 which is for Koschei only). During the duel, they come out of the shelters, exchange a glass of liquid, drink it immediately and then come back to the shelters.

Find the optimal strategy for Ivan. Note that spies of Koschei can pry into Ivan's actions! Moreover, with the help of a time machine Koschei can predict the upcoming event 10 minutes ahead of time.

This game can be downloaded from: **http://deoma–cmd.ru/en/Products/MathGames.aspx.**

Please play and try to find a way to make Ivan alive. Too hard? Read the solution.

Economic Games

Problem 2

The gamer has a pair of dice. He can throw them no more than N times. After throwing the dice n times with $n < N$, the gamer can choose to take the sum of two numbers of the dice or continue playing. If the gamer has thrown the dice N times, he has to take the sum of the dice numbers.

Find the expected value of the payoff for the game depending on N. What is N for which the gamer should not take 10? How about the N value for which the gamer should not take 11?

Problem 3

A sequence contains N random distinct numbers and the gamer can only see one number at a time. At a number provided by the game, the gamer can choose to decline it or accept it. The game is over whenever the gamer accepted a number. If the gamer has declined the first N-1 numbers, he has to accept the last number. The gamer wins if he selected the biggest number of the sequence.

Suppose that each position of the sequence has an equal chance to have the biggest number, find the value of the game (probability of winning) and the optimal strategy for the game as a function of N.