PROBLEM CORNER

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Problem 1

Consider parallelogram ABCD, $|AB| \neq |BC|$. Let E be the intersection of the perpendicular to the diagonal AC dropped from the point D with the line BC and let F be the foot of the perpendicular from the point B to the line DE. Assuming that the lines CF and AE perpendicular, determine the angle ACB.

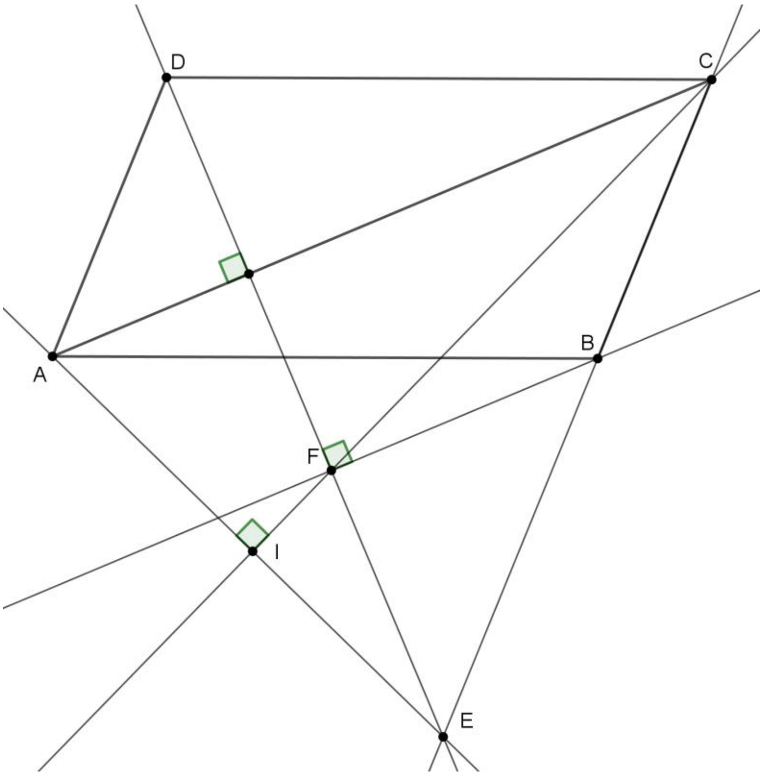

Figure 1 – Parallelogram

Problem 2

Consider a triangle ABC and its circumscribed circle k. On the circle choose an arbitrary point P and inside the triangle select an arbitrary point G. Consider circles GAB, GBC, GCA. Denoting $P_{AB},\ P_{BC},\ P_{CA}$ the inverse images of the P with respect to the circles, prove or answer following statements:

a) Points $P_{AB},\ P_{BC},\ P_{CA}$ and G lie on a circle C.

b) As P moves along the circle k, the centre of the circle C moves along a line.

c) Determine in the triangle a point $G = G_L$ in such a way that the points P_{AB}, $P_{BC},\ P_{CA}$ and G_L are always collinear (we consider a line as a special case of a circle).

Hint: Apply the Simson-Wallace theorem.

Problem 3

On a circle k are arbitrarily selected points A, B, C, D. Denote the orthocenter of the triangle ABC as H_D and analogically introduce the orthocenters $H_A\ H_B\ H_C$. Prove that the orthocenters lie on a circle with its radius equal to the radius of the circle k.