PROBLEM CORNER

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Let *Q* be a convex quadrilateral with vertices A, B, C, D.

We call edges of *Q* the four sides and the two diagonals, AB, BC, CD, DA, AC, BD.

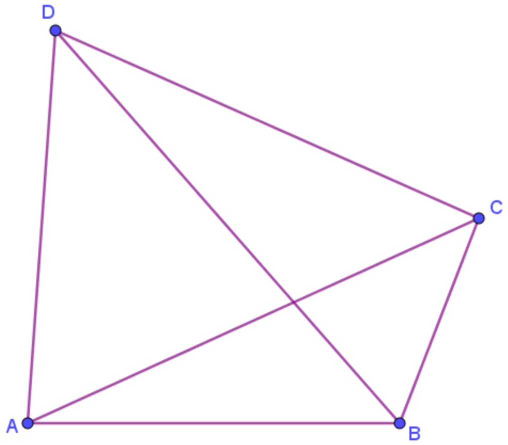

Figure 1. The quadrilateral *Q*

Problem 1

Let M_1, M_2, M_3, M_4, M_5, M_6 be the midpoints of the edges AB, BC, CD, DA, AC, BD.

Prove that the segments M_1M_3, M_2M_4, M_5M_6 are concurrent in a point G that bisects them all.

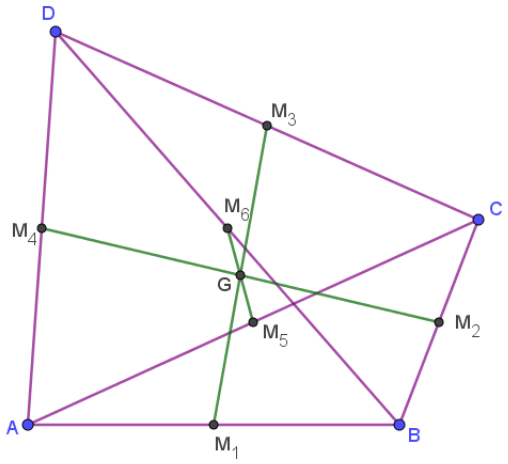

Figure 2. *Q* and the midpoint segments

Problem 2

Let A', B', C' and D' be the centroids of the triangles BCD, ACD, ABD and ABC respectively.

Prove that

- the segments AA', BB', CC' and DD' are concurrent in G;
- G divides each segment in two parts, the one containing the vertex twice the other one.

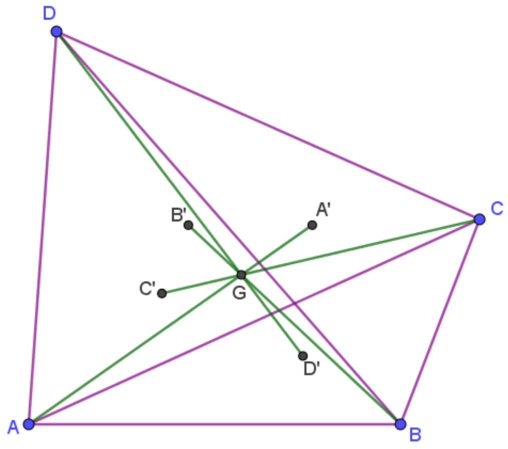

Figure 3. *Q* and centroid segments