PROBLEM CORNER

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MOTIVATION

In this Problem Corner we present two proposals coming from very different sources. Problem 1 is a generalization of the so-called Clough's conjecture, introduced and discussed by the well-known expert on mathematics education and geometry, prof. M. De Villiers[1], for the case of an equilateral triangle. The generalization is due to Alvaro Gamboa, a young, talented student (currently coursing last year of a Spanish High School=I.E.S.), participant on the Spanish Mathematical Olympiads.

On the other hand, Problem 2 has been proposed by Ricardo Barroso, a retired professor from the University of Seville, editing, since the year 2000, a Laboratorio de Triángulos Cabri https://personal.us.es/rbarroso/trianguloscabri/, i.e. a sort of Spanish version of the eJMT "Problem Corner", focusing on geometry and dynamic geometry programs, with circa 1000 proposed problems so far! We thank both of them for their contribution.

PROBLEM 1

Let $X_1, X_2, ..., X_n$ be the vertices of a regular n-gon P and let P be any point interior to P. We denote by P_{ij} the projection of P onto $r(X_i, X_j)$. By abuse, we denote $X_{n+1} := X_1$, and so $P_{n,1} := P_{n,n+1}$. Prove that the sum

$$\sum_{i=1}^{n} X_i P_{i,i+1}$$

is constant, that is, it does not depend on the point P (see Figure 1 for an example).

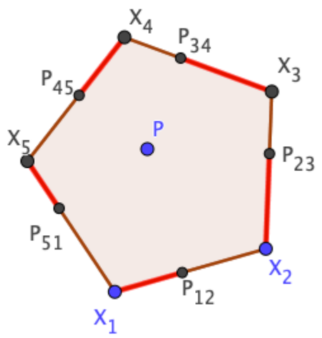

Figure 1. Case of regular pentagons: show that the sum of the red segments is independent of P

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[1]See:
• De Villiers, M., "Clough's conjecture: A Sketchpad investigation". In S. Nieuwoudt, S. Froneman, and P. Nkhoma (Eds.), Proceedings of the 10th Annual National Congress of the Association for Mathematics Education of South Africa, Potchefstroom: AMESA, July 2004, Vol. 2, pp. 52--56.
• De Villiers, M., "An illustration of the explanatory and discovery functions of proof". Pythagoras, (2012), 33(3), Art. 193, 8 pages. http://dx.doi.org/10.4102/pythagoras. v33i3.193

PROBLEM 2

Let I be the incenter of a triangle ΔABC, that is, the point of intersection of the bisectors of the angles of the triangle. Let l_1, l_2 and l_3 be, respectively, the lines which are perpendicular through I to the lines $r(A,I)$, $r(B,I)$ and $r(C,I)$. Prove that the points

$$X := r(B,C) \cap l_1,\ Y := r(A,C) \cap l_2 \text{ and } Z := r(A,B) \cap l_3.$$

are collinear.