PROBLEM CORNER

Provided by Jen-chung Chuan

E-mail: jcchuan@gmail.com

Interactive solutions can be found at this http://sylvester.math.nthu.edu.tw/d2/3problems/2014-9/PROBLEM%20CORNER.html

Problem 1

Let the sequence of integers $\{a(r.s) : s = 1,2,...,2^{r-1}, r = 1,2,...\}$ be given by *a(1,1)=1*,

a(r+1,s) = a(r, (s+1)/2) + 1 if s is odd, else *a(r+1,s) = a(r+1,s-1)*a(r, s/2) + 1*.

Show that the sum of reciprocals

1/a(r,1) + 1/a(r,2) + ... + 1/a(r, 2^{r-1}*))* converges to π/4 as *r* approaches to infinity.

Solution to Problem 1. Provided by the poster session of the 2014 Taiwan delegate Wang Kuan-Yu, age 13, at International Science and Engineering Fair (Intel ISEF). He eventually received 4th Award. The solution can be found at this link: http://mathandtech.org/eJMT_Prob_Corner_June14/Solution_Problem1.pdf.

Problem 2

Construct 24 circles each touching exactly four others. (In space or on a plane, it doesn't matter.)

Solution to Problem 2. The solution using Cabri plugin can be found at this link: http://sylvester.math.nthu.edu.tw/d2/3problems/2014-9/Problem%202.html, and the corresponding file can be found at this link: http://mathandtech.org/eJMT_Prob_Corner_June14/Solution_Problem2.cg3.

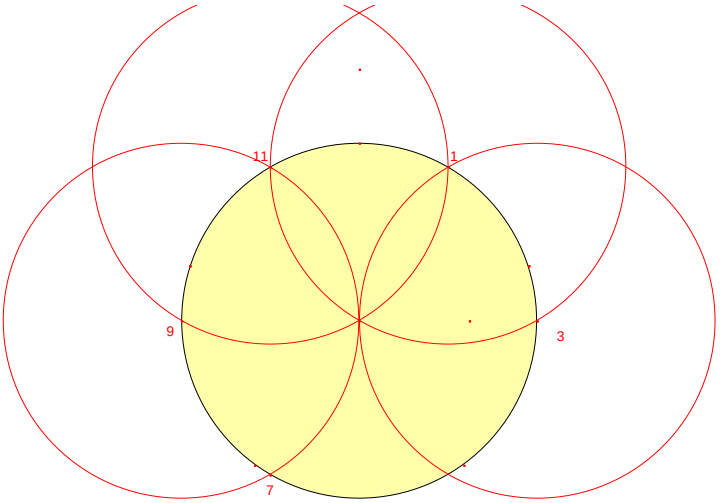
Problem 3

Construct five points forming the vertices of a regular pentagon using compass only.

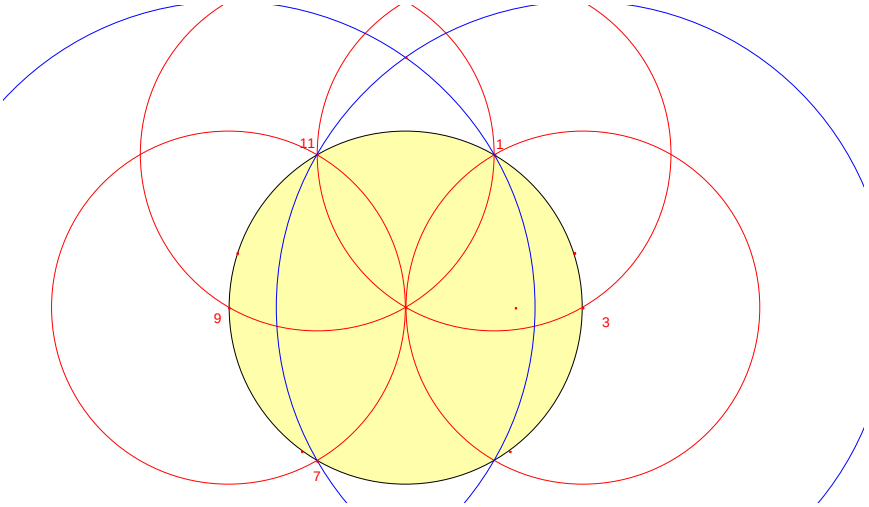
Solution to Problem 3. [Note. 1. Solution using Cabri II plugin can be found at this link: http://sylvester.math.nthu.edu.tw/d2/3problems/2014-9/compass.html; 2. Corresponding Cabri II plus file can be found http://sylvester.math.nthu.edu.tw/d2/3problems/2014-9/compass.html.fig.]

Given the filled circle of radius 1, we are to construct five points comprising the vertices of a regular pentagon.

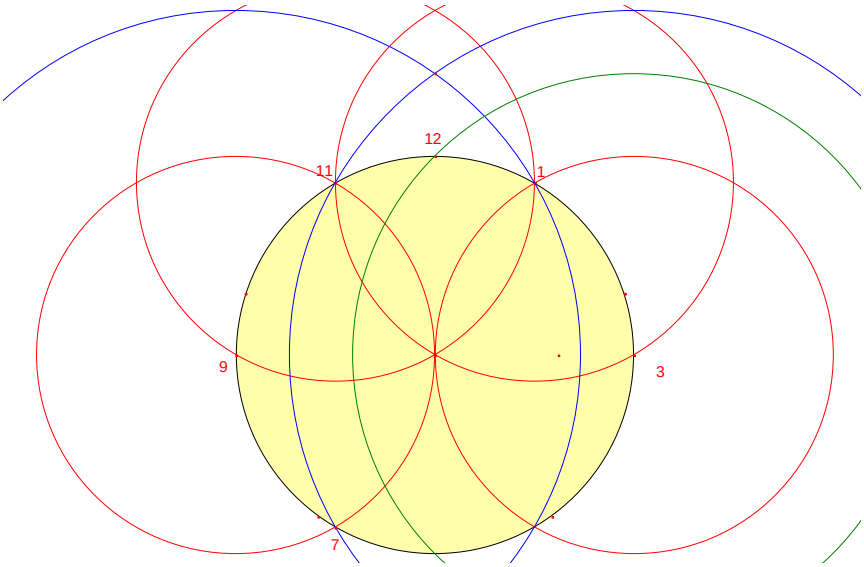
Step 1: Starting from 3 o'clock position, find 1 o'clock, 11 o'clock, 9 o'clock, 7 o'clock positions with the red circles.



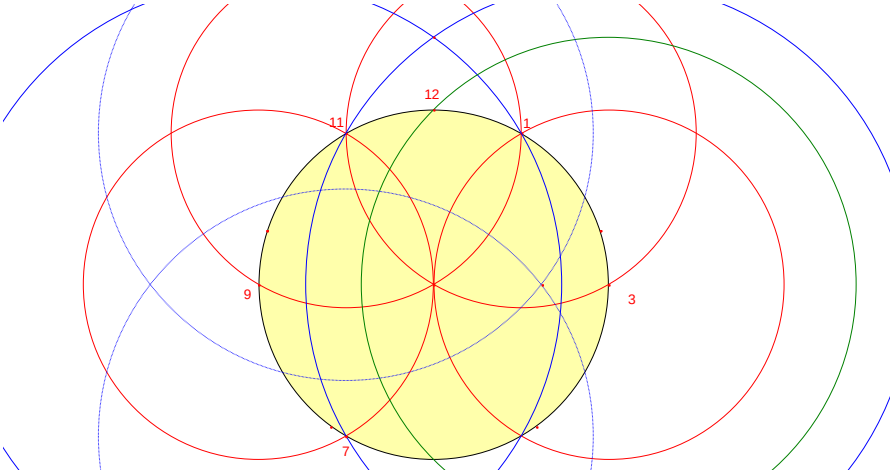
Step 2: Draw blue circles (of radius \root 3) meeting at points \root 2 away from the center.



Step 3. Locate the 12 o'clock position with the green circle with center at 3 o'clock position and radius \root 2.

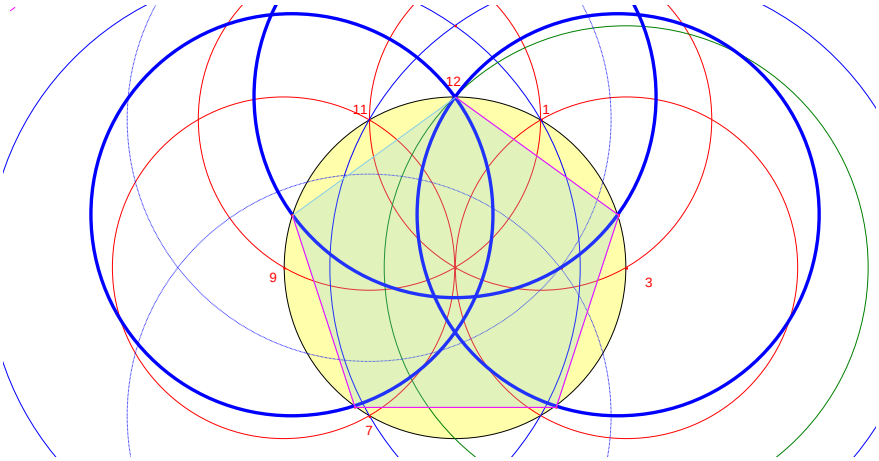


Step 4: Draw dotted circles with centers at 11 and 7 radius \root 2



Step 5: Draw the thick circles with center at 12 passing through the point of intersection of two dotted circles inside the given circle. This way, the thick circles will construct vertices of regular pentagon having 12 as a vertex.

Reference: Courant and Robbins, What is Mathematics?


12
11
1
9
3
7