Solutions to Problem Corner-June 2016

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Problem 1

A series is just an infinite sum. It can be defined as the limit when n goes to infinity of the nth partial sums (that is, the sum of the first n terms). In this problem we consider the sum of a series in which the coefficients satisfy a linear recurrence. Compute the sum of the series

$$\sum_{n=0}^{\infty} a_n x^n \, ,$$

where the sequence (a_n) satisfy $a_0 = 1$, $a_1 = 2$ and

$$5a_n + 2a_{n-1} - 4a_{n-2} = 0$$

(Answer: $(5 + 12x)/(5 + 2x - 4x^2)$). Link to RecurrentSeries.pdf

Problem 2

Infinite products, first studied by Euler and developed extensively by Weierstrass, can be defined analogously to series, as the limit of their partial products. Compute the infinite product

$$\prod_{n=0}^{\infty} \frac{a^{2^n} + 1}{a^{2^n}}$$

where $a > 1$ (Answer: $a/(a - 1)$). Link to InfiniteProducts.pdf

Remark: While these problems can be done "by hand" (none of them requires maths beyond elementary calculus of limits), it is much more interesting to do some experimentation first with a CAS (such as Maxima) to get an idea of the relations and patterns between the inputs of the problems, which can lead to educated guesses for the solutions.