PROBLEM CORNER

Provided by Weng Kin Ho
National Institute of Education, Nanyang Technological University, Singapore
wengkin.ho@nie.edu.sg

Problem 1
There are 17 rooms in a row with a door between every two adjoining rooms. Additionally, each room has a door opening to the corridor. A princess lives in one room each day and moves to an adjoining room the next day at 7 in the morning. A prince wants to talk to the princess and knocks on one door each day at noon. If the princess is in that room, she will open the door and they can converse. Otherwise, the prince leaves and tries again at some door at the same time the next day. Is there a strategy to guarantee that the prince will meet the princess if he has 30 days to try?

Solution
Number the rooms from 1, 2, 3, …, through to 17.
1 2 3 4 … … … … 16 17
Each room number is either odd or even.
Assume without loss of generality that the room number of the room in which the princess is at on Day 1 is an even number.
Case 1. You selected a room on Day 1 whose number is even.
Then the princess and you are of the same odd-even parity for all the days that follow. So you just comb through the rooms from 2 to 16. If you do not meet the princess within these 15 days, you just need to turn back and comb through rooms till you reach Room 2. Since you and the princess has the same odd-even parity, you will meet the princess one of these 29 days.
Case 2. You selected a room on Day 1 whose number is odd.
Then the princess and you are of opposite odd-even parity for all the days that follow. So you just comb through the rooms from 2 to 16. But this time, if you do not meet the princess within these 15 days, you still knock at Room 16 on Day 16. Since you can change the odd-even parity between you and the princess (but not the princess!), then you will be certain that the princess is in an even-numbered room on Day 16. Now you turn back and head towards Room 2, knocking at each of the doors from 15, 14, …, to 2. You will be guaranteed to meet the princess in the worst-case scenario on Day 30.

Problem 2
A horizontal meter ruler has 7 ants randomly dropped on it. Once on the ruler, each ant chooses to crawl to the right or to the left, independent of other ants, at a constant speed of 1 meter per minute and does not change

direction unless it meets another ant coming towards it, whereupon it changes its direction but not its speed. When an ant reaches either end of the ruler, it falls off the ruler. Find out the least time duration since the commencement of the ants' motion till all ants fall off the ruler.

Solution

Imagine two ants meet, and each one turns in the opposite direction. Another equivalent way to view this is that you may treat all ants as indistinguishable, i.e., each ant did not turn but instead continues its journey in the same direction. Thus, the least time you need to wait is for the worse-case scenario of having an ant travelling the full length of the one-meter ruler.