Sudoku-Part 1 (Exerpt of the author's Chinese book "Completely Cracking Sudoku")

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Maze with clues has been built in every foreseeable place,
All barriers could be removed without any frustrating face;
In idle time please come to visit the three treasures palace,
Relax your mood and nerves and indulge in Sudoku space.

The author graduated from Department of Mathematics, National Taiwan Normal University and Institute of Mathematics, National Tsinghua University, received M.S. from University of Wisconsin-Milwaukee and Ph.D. from University of Illinois. He had nearly twenty years of teaching which was interrupted by eight years of actuarial career and retired from San Francisco State University in 2002.

He has published three textbooks: College Mathematics, Actuarial Mathematics Made Simple (in Chinese) and Management Mathematics Made Simple (in Chinese); one semi-autobiography: "From 1949 to 2011" and one technical knowhow: "Completely Cracking Sudoku" (both in Chinese); two lecture notes: "Mathematics as a Creative Art" (translated from the lecture notes of Paul Halmos) and "General View of Actuarial Mathematics." He has also published thirty academic papers, one hundred pieces of prose and poems (in Chinese), seven hundred political commentaries (in Chinese).

Foreword

Figure A

6_{14}	5*	7*	1*	8_{18}	9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*
4*	8_{15}	2*	3*	6_{19}	7*	9*	5*	1*
3_{12}	6_{13}	9*	2*	1_{1}	4*	5*	8_{3}	7*
8_{16}	7_{17}	1*	9_{7}	3_{6}	5*	6*	2_{2}	4*
5*	2_{4}	4*	6*	7_{5}	8*	3*	1*	9*
2*	4*	6*	5*	9_{9}	1*	8*	7_{10}	3*
1*	3*	8*	7_{8}	4*	6*	2*	9_{11}	5*
7*	9*	5*	8*	2*	3*	1*	4*	6*

Su Doku is the Japanese abbreviation of "Independent Number Place". Its original name was "Number Place." It traced back to the Latin square (as shown in Figure A, not including stars and subscripts) invented by the eighteenth century Swiss mathematician Leonhard Euler. If we conceal all unstarred numbers, then we convert it back to a puzzle form (as shown in Figure 1).

Figure 1

	5*	7*	1*		9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*
4*		2*	3*		7*	9*	5*	1*
		9*	2*		4*	5*		7*
		1*			5*	6*		4*
5*		4*	6*		8*	3*	1*	9*
2*	4*	6*	5*		1*	8*		3*
1*	3*	8*		4*	6*	2*		5*
7*	9*	5*	8*	2*	3*	1*	4*	6*

The purpose of the Sudoku game is using logical inference, starting from the puzzle form of Figure 1, to uncover those unstarred numbers in Figure A step by step according to the order of subscripts. The rule of Sudoku game is to require each row, each column and each box to have each of all numbers from 1 through 9.

The inventor of Sudoku games was Tetsuya Nishio, who later started actively promoting new variations of Sudoku in 2007. He first came across a game named Number Place in Dell Magazine in early 1980's while visiting the U.S. and then developed it into a more complicated puzzle to be played in Japan. Its name was immediately changed to Sudoku by Nikoli Magazine in Japan and prevailed there for a while. People all over the world are now indulging in this game thanks to Wayne Gould, a retired Hong Kong judge from New Zealand. Not until 1997 while touring Tokyo, he encountered this gadget. After six years of study, he came up with the computer software named Pappocom which enabled him to massively produce fiendish Sudoku puzzles. In 2004, this wonderful workmanship game frantically hit the entire England and subsequently the whole Europe. Soon after that, it returned to U.S. and Japan, further extended to Taiwan in 2005. Surging from the outset of this century, "Sudoku" is indeed self-entertaining, time-killing, lonliness-removing, solitude-exempting and senile-preventing.

Freshly retired from the teaching post of San Francisco State Business School, I started to play this game sporadically. No sooner than 2005, the returning year of my son Michael from medical training, I began to indulge myself in this fascinating game, thanks to his thoughtful choices of all sorts of challenging Sudoku books as birthday, father day and Chistmas presents for the subsequent three years, including 1001 SUDOKU (Thunder's Mouth Press, copy right to Nicoli) and SUDOKU GENIUS (Tom Scheldon, 144 of the Most Friendish Puzzles Ever Devised) of 2005; Su Doku (Wayne Gould, Challenging Sudoku 4), HIGHER SUDOKU (Tetsuya Nishio, New Variations from Japan) and Sudoku Puzzles (Aline Ribeiro de Almeida,TOP 100 HARDEST) of 2006; Extreme Sudoku (Dell, Sudoku puzzles with an X factor!) of 2007. Therefore, I literally ate and drank Sudoku during the entire period of those three years. However, unlike most speed-oriented players, I took my time to enjoy the logical reasoning provided by each puzzle and kept the detailed record of the whole solving process. The joy of life is to share. With this belief, I had prepared a draft of my book "Completely Cracking Sudoku" way back in 2007 blending the most inspiring ideas of puzzle structures inlighted by the afore-mentioned books in order to introduce the unique step by step method. The key is to take and record each step in accordance with a logical reasoning instead of hasty trials and errors, so that everyone can enjoy and refresh one's memorable moments.

That draft was then sent to my youngest brother Yung-Shyeng who never played a single game of Sudoku. He made lots of valuable suggestions from a begginer's point of view. He also added a finishing touch, liking of the secrete codes in kungfu practice, on this originally scrupulous and methodical manuscript of knowhow. This has revivd the spirit of my book as if bringing the painted dragon to life by putting in the pupils of its eyes. Soon after that, I was sidetracked by my breakthrough in the classic number theory. Coincidentally, the afore-mentioned Euler was a famous classic number theorist, who along with Gauss, Bernoulli and Stirling had almost simultaneously discovered various formulas for expressing the sum of powers of the natural sequence. Imaging that, had he had spare time to spend on Latin squares, Sudoku games could have come about some three hundred years ago! As to my breakthrough, I generalized most of those formulas from the natural sequence to arithemetically progressive senquences and obtained their polynomial expressions.

Just around the conclusion of my breakthrough, I was informed by Mr. Ray Leo in early July of 2012 that the hardest Sudoku was newly posted online. After being able to crack down this hardest Sudoku in a couple of days using my Sudoku-solving techniques, I have revived the desire of publishing my book. During this five years of "idiling period", I have actually perfected the method of explaining how puzzles can be solved step by step using various techniques with the aid of shorthand annotations to be introduced in my book. In fact, most of so called challenging puzzles turned out to be so so under the scrutiny of my examination. Nevertheless, they more or less reflected those authors' special view points and therefore should not be categorically denied.

Interestingly, in 2008 I picked up and studied "Cracking Sudoku" (in Chinese, by Wang Tung Chiao) while strolling the "Bookstore Street" in Taipei. The following year, I have pointed out an erroneous puzzle of Will Shortz's THE DANGEROUS BOOK OF SUDOKU and received three of his new books in return. So it is fair to say that I have not given up on Sudoku completely. Thus in the final section of this article, we shall let readers take part in solving the hardest Sudoku to manifest what they are about to learn is by no means a “flowery boxing“. Furthermore, we might as well let veterans peek at a few puzzles from the above two books now so that they can forsee what would unfold in the later sections, for fear that they might give up on this article due to the unchallenging nature of the first few sections. Although most puzzles we shall encounter were labeled as rank 5, they could be solved rather easily with patience and perseverance; even the beginners could follow the step by step guidance and enjoy the wonderful feeling. Otherwise, they can skip this foreplay and come back to visit these puzzles after learning the basic skills. To begin with, let us try the most challenging puzzle claimed by Wang Tung Chiao in Cracking Sudoku.

Foreplay

We first star all given numbers in the figure and then start with the smallest number ready to be filled, according to the prescribed order of up-down and left-right. After failing with 1, 2 and 3 for all boxes, we try 4 in box 1. Grid (12), the intersection of row 1 and column 2, is the only place for 4, abbreviated as 4(12), because of 4(41), 4(73) and 4(37). So the first step is 4_1(12).

The second step is to enter 4 into the grid of row 8 and column 9 in box 9, abbreviated as 4_2(89) and the third step is to enter 5 into the grid of row 1 and column 7 in box 7, abbreviated as 5_3(17).

1*	4_1			9*		5_3		3*
	7*			5*			6*	
		2*	8*		1*	4*		
4*								5*
		6*				7*		
9*								8*
		4*	5*		9*	2*		
	3*			6*			5*	4_2
2*				4*				6*

Next, looking around from left to right, we can no longer find a free lunch as before, meaning that we encounter the first obstacle.

Don’t be discouraged. With patience and perserverence, we might find the grid in row 1 and column 3, but what number to fill in? Please scan from left to right, row 1 has 1, 4, 9, 5, 3 and column 3 has 2, 6, 4, hence only 7 and 8 are left to be filled. But, wait! 7 can not be filled here either, due to the fact that box 1 where the grid in question is situated has 7.

1*	4_1	8_4		9*		5_3	2_7	3*
	7*	9_9		5*		8_5	6*	1_6
		2*	8*		1*	4*	7_{14}	9_{13}
4*								5*
		6*				7*		2_8
9*								8*
		4*	5*		9*	2*		7_{15}
	3*			6*		9_{12}	5*	4_2
2*	9_{10}	5_{11}		4*				6*

Hence for the fourth step, we can take 8_4(13) as shown in the above figure. This is called a grid move (g), abbreviated as 8_4(13)g, since this move is determined by the surroundings (row, column & box) intersecting with this grid. After 8_5(27) and 1_6(29), we can look at box 7. The 2 can only be entered into (18), abbreviated as 2_7(18)b7. This is called a box move (b), since this move is determined by the surroundings (all rows & columns) intersecting with this box.

After 2_8(59), we can look at row 2. The 9 can only be entered into (23), abbreviated as 9_9(23)r2. This is called a row move (r), since this move is determined by the surroundings (all columns &

boxes) intersecting with this row. After 9_{10}(92), 5_{11}(93) and 9_{12}(87), we can look at column 9. The 9 can only be entered into (39), abbreviated as 9_{13}(39)c9. This is called a column move (c), since this move is determined by the surroundings (all rows & boxes) intersecting with this column. After 7_{14}(38) and 7_{15}(79), we once again encounter a stalemate.

Naturally, we might expect a grid move coming to rescue, since it is the most complicated move (involving intetsecting row, column & box) among all of the moves discussed above. Where could this grid move be? By scanning three unfilled grids in box 1, we can easily know to fill 3 into (21), abbreviated as 3_{16}(21)g as shown in the figure.

We can then move rather smoothly by taking 3_{17}(35), 3_{18}(78)r7, 8_{19}(98) , 1_{20}(97) and 7_{21}(81)c1 as shown.

1*	4_{1}	8_{4}		9*		5_{3}	2_{7}	3*
3_{16}	7*	9_{9}		5*		8_{5}	6*	1_{6}
		2*	8*	3_{17}	1*	4*	7_{14}	9_{13}
4*								5*
		6*				7*		2_{8}
9*								8*
		4*	5*		9*	2*	3_{18}	7_{15}
7_{21}	3*			6*		9_{12}	5*	4_{2}
2*	9_{10}	5_{11}		4*		1_{20}	8_{19}	6*

The rest is left for the readers to complete, but we must warn you that grid moves might be needed in several occasions! We shall provide a step by step solution in the following figure, with some helpful anottations of row, column and grid moves.

1_{22}(83)g

6_{38}(67)g

8_{24}(55)g

6_{40}(46)g

2_{32}(62)c2

9_{46}(58)r5

2_{34}(45)c5

1*	4_{1}	8_{4}	6_{41}	9*	7_{42}	5_{3}	2_{7}	3*
3_{16}	7*	9_{9}	4_{48}	5*	2_{49}	8_{5}	6*	1_{6}
6_{28}	5_{29}	2*	8*	3_{17}	1*	4*	7_{14}	9_{13}
4*	8_{25}	7_{36}	9_{54}	2_{34}	6_{40}	3_{39}	1_{55}	5*
5_{30}	1_{33}	6*	3_{45}	8_{24}	4_{47}	7*	9_{46}	2_{8}
9*	2_{32}	3_{37}	1_{53}	7_{35}	5_{31}	6_{38}	4_{52}	8*
8_{26}	6_{27}	4*	5*	1_{23}	9*	2*	3_{18}	7_{15}
7_{21}	3*	1_{22}	2_{51}	6*	8_{50}	9_{12}	5*	4_{2}
2*	9_{10}	5_{11}	7_{43}	4*	3_{44}	1_{20}	8_{19}	6*

As we have experienced, only the basic moves are sufficient to solve the above puzzle without resorting to those “unique nine tricks“introduced in that book.

How about the second most challenging puzzle in the same book as shown below? Try it out!

	1*			3*				
		6*			2*			
	9*				4*	8*		
		7*					1*	4*

1*		5*				3*		8*
4*	2*					5*		
		2*	9*				6*	
			4*			7*		
				7*			2*	

After the first nine moves:

2_{1}(85) 6_{6}(69)b8

4_{2}(55) 6_{7}(17)

8_{3}(88) 3_{8}(33)g

2_{4}(47)b8 6_{9}(52)g

2_{5}(54) ,

you'll come across the stalemate as shown in the next figure:

	1*			3*		6_{7}		
		6*			2*			
	9*	3_{8}			4*	8*		
		7*				2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}		3*		8*
4*	2*					5*		6_{6}
		2*	9*				6*	
			4*	2_{1}		7*	8_{3}	
				7*			2*	

As for the next step, since column 8 of box 8 has a hidden 7, we can take 5_{10}(38)g:7c8b8 to relieve the temporary hardship.

After the next twelve steps:

3_{11}(28)c8 1_{15}(83) 8_{19}(13)
4_{12}(18)c8 4_{16}(77)
9_{20}(58)c8
4_{13}(22) 7_{17}(72)c2 7_{21}(56)
4_{14}(93) 9_{18}(63)c3 7_{22}(68) ,

we shall encounter a real touchy problem as shown in the figure.

	1*	8_{19}		3*		6_{7}	4_{12}	
	4_{13}	6*			2*		3_{11}	
	9*	3_{8}			4*	8*	5_{10}	
		7*				2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}				5*	7_{22}	6_{6}
	7_{17}	2*	9*			4_{16}	6*	
		1_{15}	4*	2_{1}		7*	8_{3}	
		4_{14}		7*			2*	

7	1*	8_{19}	5	3*	9	6_{7}	4_{12}	2
5	4_{13}	6*	1	8	2*	9	3_{11}	7

All logical reasonings seem to fail momentarily, we could not help but resorting to the instinct. As a matter of fact, it is not all that bad as demonstrated in the figure.

Warning: you are now stepping into veterans' jungle – you may follow the instruction to complete the entire journey just to get a sense of what's going on, and revisit later anytime you choose to.

2	9*	3_{8}	7	6	4*	8*	5_{10}	1
3	8	7*	6	9	5	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}	3	1	8	5*	7_{22}	6_{6}
8	7_{17}	2*	9*	5	1	4_{16}	6*	3
		1_{15}	4*	2_{1}		7*	8_{3}	
		4_{14}	8	7*		1	2*	

Feeling just like drifting with a flow, what a feat! Why could we not have such great feelings before? The answer is about to be disclosed, please take a look at this and the next two figures.

7	1*	8_{19}	5	3*	9	6_{7}	4_{12}	2
5	4_{13}	6*	1	8	2*	9	3_{11}	7
2	9*	3_{8}	7	6	4*	8*	5_{10}	1
3	8	7*	6	9	5	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}	3	1	8	5*	7_{22}	6_{6}
8	7_{17}	2*	9*	5	1	4_{16}	6*	3
6	5	1_{15}	4*	2_{1}	3	7*	8_{3}	9
9	3	4_{14}	8	7*	6	1	2*	5

7	1*	8_{19}	5	3*	9	6_{7}	4_{12}	2
5	4_{13}	6*	1	8	2*	9	3_{11}	7
2	9*	3_{8}	7	6	4*	8*	5_{10}	1
3	8	7*	6	9	5	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}	3	1	8	5*	7_{22}	6_{6}
8	7_{17}	2*	9*	5	1	4_{16}	6*	3
9	3	1_{15}	4*	2_{1}	6	7*	8_{3}	5
6	5	4_{14}	8	7*	3	1	2*	9

These two figures indicate that the puzzle in question has multiple solutions – Please pay attention to the last two rows, several numbers are flipped between the two figures.

2_{31}	1*	8_{19}	7_{33}	3*	5_{34}	6_{7}	4_{12}	9_{23}
5_{30}	4_{13}	6*	8_{25}	9_{24}	2*	1_{27}	3_{11}	7_{29}

To show you more problems with this puzzle, let us go back to the original stalemate and start with 9_{23}(19). Then we would be led to a new stalemate in the following figure.

7_{32}	9*	3_{8}			4*	8*	5_{10}	2_{28}
		7*			9_{38}	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}				5*	7_{22}	6_{6}
	7_{17}	2*	9*			4_{16}	6*	
9_{35}		1_{15}	4*	2_{1}	6_{37}	7*	8_{3}	
6_{36}		4_{14}		7*		9_{26}	2*	

Now by taking the liberty to start with 3_{39}(41), we would be led to the following dilemma. Had we enter 1 into (79), then 3, 5 in boxes 3 and 9 would be interchangeable for the puzzle to have two pairs of solutions.

2_{31}	1*	8_{19}	7_{33}	3*	5_{34}	6_{7}	4_{12}	9_{23}
5_{30}	4_{13}	6*	8_{25}	9_{24}	2*	1_{27}	3_{11}	7_{29}
7_{32}	9*	3_{8}			4*	8*	5_{10}	2_{28}
3_{39}	8_{40}	7*			9_{38}	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}		8_{43}		5*	7_{22}	6_{6}
8_{41}	7_{17}	2*	9*			4_{16}	6*	1?
9_{35}	3/5	1_{15}	4*	2_{1}	6_{37}	7*	8_{3}	5/3
6_{36}	5/3	4_{14}		7*	8_{42}	9_{26}	2*	3/5

Therefore, we should take 1_{44}(99) as in the following two figures.

2_{31}	1*	8_{19}	7_{33}	3*	5_{34}	6_{7}	4_{12}	9_{23}
5_{30}	4_{13}	6*	8_{25}	9_{24}	2*	1_{27}	3_{11}	7_{29}
7_{32}	9*	3_{8}	6	1	4*	8*	5_{10}	2_{28}
3_{39}	8_{40}	7*	5	6	9_{38}	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}	1	8_{43}	3	5*	7_{22}	6_{6}
8_{41}	7_{17}	2*	9*	5	1	4_{16}	6*	3
9_{35}	3	1_{15}	4*	2_{1}	6_{37}	7*	8_{3}	5
6_{36}	5	4_{14}	3	7*	8_{42}	9_{26}	2*	1_{44}

2_{31}	1*	8_{19}	7_{33}	3*	5_{34}	6_{7}	4_{12}	9_{23}
5_{30}	4_{13}	6*	8_{25}	9_{24}	2*	1_{27}	3_{11}	7_{29}
7_{32}	9*	3_{8}	1	6	4*	8*	5_{10}	2_{28}
3_{39}	8_{40}	7*	6	5	9_{38}	2_{4}	1*	4*
1*	6_{9}	5*	2_{5}	4_{2}	7_{21}	3*	9_{20}	8*
4*	2*	9_{18}	3	8_{43}	1	5*	7_{22}	6_{6}
8_{41}	7_{17}	2*	9*	1	3	4_{16}	6*	5
9_{35}	5	1_{15}	4*	2_{1}	6_{37}	7*	8_{3}	3
6_{36}	3	4_{14}	5	7*	8_{42}	9_{26}	2*	1_{44}

However, the solution provided in that book is yet different from the above four! What does all this indicate? This puzzle was so hastily devised that plenty of time had been wasted by players, as a result how many people would still want to try the rest of puzzles in that book?

Nevertheless, we should not totally deny that book with one bad example, since even the famous Will Shortz made mistakes as illustrated below.

5	1*	8	7	3*	9	6	4	2
7	4	6*	8	5	2*	9	3	1
2	9*	3	1	6	4*	8*	5	7
8	3	7*	5	9	6	2	1*	4*
1*	6	5*	2	4	7	3*	9	8*
4*	2*	9	3	1	8	5*	7	6
3	7	2*	9*	8	1	4	6*	5
6	5	1	4*	2	3	7*	8	9
9	8	4	6	7*	5	1	2*	3

The following is the very last puzzle in Will's book, the work of a master hand is indeed not a common chord because it took me, according to a Chinese idiom, "the strength of nine bulls and two tigers" to barely crash through…

	7*							
	3*					1*	2*	
		9*	6*	5*				
5*		8*	9*	6*				
				1*			5*	
	6*		4*					
4*		2*			8*			5*
		7*				8*	1*	
					3*	7*		

	7*		(8/3)	(3/8)		5_{2}		
(8)	3*	5_{3}				1*	2*	

After taking five consecutive steps as in the figure with the ending box move, we come across a difficult barrier.

Only 6 or 8 can be filled in (21). Had we entered 8, we would end up with the dilemma that no 2 could be filled in box 4. Hence we can take 6_6(21) as in the following figure.

		9*	6*	5*				
5*		8*	9*	6*	2/7			
			3/8	1*	7/2		5*	
	6*		4*	8/3	5_1			
4*		2*			8*			5*
3_5		7*			6_4	8*	1*	
					3*	7*		

Just managed to take 6_7(93), we immediately come across a very touchy problem, so called “inch step no way” in Chinese (literary, hard to inch forward).

In the jungle one cannot take the liberty on one’s own body. Let’s take a serious look at column 3 to see what’s going on.

Only 1 or 4 can be placed at (13).

	7*	(1_1)		(3_9)		5_2		
6_6	3*	5_3				1*	2*	
	(4_4)	9*	6*	5*				
5*		8*	9*	6*	2/7			
		(4_2)	(3_8)	1*	7/2		5*	
	6*	(3_3)	4*		5_1			
4*		2*			8*			5*
3_5	(5_6)	7*	(2_7)		6_4	8*	1*	
	(8_5)	6_7			3*	7*		

Had we placed 1, the nineth step would lead to the dilemma that 2 coould not be entered into box 4. Therefore, we have

4_8(13)g：
1(13)→4(53)→3(63)

	7*	4_8	3_{14}	2_{15}		5_2		
6_6	3*	5_3	7_{17}	8_{16}		1*	2*	
		9*	6*	5*				
5*		8*	9*	6*				1_{11}
		3_{10}	8_{13}	1*			5*	
	6*	1_9	4*	3_{12}	5_1			
4*	9_{19}	2*	1_{20}	7_{18}	8*			5*
3_5		7*			6_4	8*	1*	
		6_7			3*	7*		

→4(32)→8(92)
→5(82)→2(84)g
→3(54)→3(15)

→no2b4

9_{19}(72)r7:36r7b9

After cruising up to the twentieth step, the next is a grid move. Where is it? Please wait till the next figure for solution.

Grid (82) can only be filled with 5, we have 5_{21}(82)g. Cruising till the twenty-fourth step, we are facing with yet another problem.

Where should 2 be filled in row 3?

(1/8)	7*	4_{8}	3_{14}	2_{15}		5_{2}		
6_{6}	3*	5_{3}	7_{17}	8_{16}		1*	2*	
(8/1)	(2)	9*	6*	5*				
5*		8*	9*	6*				1_{11}
		3_{10}	8_{13}	1*			5*	
	6*	1_{9}	4*	3_{12}	5_{1}			
4*	9_{19}	2*	1_{20}	7_{18}	8*			5*
3_{5}	5_{21}	7*	2_{23}		6_{4}	8*	1*	
(no#)		6_{7}	5_{22}		3*	7*		2_{24}

If we took 2(32), then the hidden 1and 8 in column 1 of box 1 would cause no number to be entered into (91). Hence we can take 2_{25}(31)r3: 2(32)→18c1b1→no#(91).

After 2_{25}(31) and 2_{26}(67) had we place the 4 of box 4 at (36), then we would run into the dilemma of not having 3 in box 8. Hence we have

	7*	4_{8}	3_{14}	2_{15}		5_{2}		
6_{6}	3*	5_{3}	7_{17}	8_{16}		1*	2*	
2_{25}		9*	6*	5*	(4_{1})	(3_{2})		
5*		8*	9*	6*				1_{11}
		3_{10}	8_{13}	1*			5*	
	6*	1_{9}	4*	3_{12}	5_{1}	2_{26}		
4*	9_{19}	2*	1_{20}	7_{18}	8*		(3_{3})	5*
3_{5}	5_{21}	7*	2_{23}		6_{4}	8*	1*	
		6_{7}	5_{22}		3*	7*		2_{24}

4_{27}(26)b4:4(36)→3(37)g→3(78)→no3b8

1_{31}	7*	4_{8}	3_{14}	2_{15}	9_{29}	5_{2}	6_{47}	8_{48}
6_{6}	3*	5_{3}	7_{17}	8_{16}	4_{27}	1*	2*	9_{28}
2_{25}	8_{32}	9*	6*	5*	1_{30}	4_{52}	7_{55}	3_{54}
5*	2_{43}	8*	9*	6*	7_{41}	3_{51}	4_{53}	1_{11}
7_{40}	4_{44}	3_{10}	8_{13}	1*	2_{42}	9_{45}	5*	6_{46}
9_{39}	6*	1_{9}	4*	3_{12}	5_{1}	2_{26}	8_{56}	3_{57}

March on thereafter without any resistance, only resorting to basic moves is sufficient to handle a butcher’s cleaver effortlessly.

4*	9_{19}	2*	1_{20}	7_{18}	8*	6_{49}	3_{50}	5*
3_{5}	5_{21}	7*	2_{23}	9_{37}	6_{4}	8*	1*	4_{38}
8_{34}	1_{33}	6_{7}	5_{22}	4_{36}	3*	7*	9_{35}	2_{24}

That book indeed lives up to its fame, full of bundles of barriers, bristled with thorns everywhere. However after clearing ten more barriers, we nevertheless come across a falsely disguised puzzle as shown below.

The first sixteen steps are rather straightforward except perhaps

3_2(61)c1 2_6(46)r4 7_7(64)r6
7_8(52)c2 9_9(92)c2 8_{16}(67)r6

Next, 4 and 5 of column 5 can only be placed at (15) and (25). Hence we obtain 16(35)/(65) as shown.

On the other hand, we also see 16(38)/(68) in column 8 so that 1 and 6 are interchangeable to force double solutions.

7*				4/5	9_{13}		8*	
9*				5/4				
2_{1}		4*		6/1	7*	9_{14}	1/6	5*
6*			4*	9*	2_{6}	3_{3}	5_{5}	7*
5*	7_{8}	9_{10}		3*		4*		
3_{2}	4_{4}	2*	7_{7}	1/6	5*	8_{16}	6/1	9_{15}
	3*		9_{11}	2_{12}	6*			
	2*			8*			9*	
	9_{9}			7*		2*	3*	

Structures

This article contains nine sections. The first four sections are intended to train beginners into veterans, the next section will use the efficient method of revealing all possible candidates for each grid and the subsequent two sections will provide readers hands-on puzzles to play with and their detailed solutions by way of shorthand annotations. We shall take readers to try out various challenging masterpieces and then lead them to solve the afore-mentioned hardest sudoku ever in the last two sections. In each of the first four sections, we shall first demonstrate one puzzle for beginners, then another more sophiscated one for veterans. At the end of each of the first three sections, we shall provide another puzzle for beginners to practice and to be discussed in the subsequent section. In such a way of rotationary training, the readers’ level of skills will be raised unnoticeably during the moments of unlimited enjoyment!

c1	**c2**	**c3**	**c4**	**c5**	**c6**	**c7**	**c8**	**c9**
	b1			b4			b7	

	b2			b5			b8	
	b3			b6			b9	

As shown in the figure, the **r1**

playground of Sudoku is **r2**

divided into 81 grids, to be **r3**

combined horizontally as **r4**

nine rows top-down, **r5**

vertically as nine columns **r6**

left-right, and 3x3 squares as **r7**

nine boxes. We follow the **r8**

prescribed order of up-down **r9**

and left-right, so the referral of each grid will be row first column next; for instance grid (32) stands for the grid located at the intersection of row 3 (r3) and column 2 (c2).

Similarly, the order of boxes is the same: box 1, box 2, box 3, box 4, box 5, box 6, box 7, box 8 and box 9 are called top-down and then left-right respectively as b1, b2, b3, b4, b5, b6, b7, b8 and b9. We shall later use the same prescribed order to place numbers at grids in rows, columns or boxes. The reason for doing so is simply to facilitate our explanations and mutual understanding with readers, but by no means to limit your flexibility in manipulation!

Our unique invention is to combine three consecutive boxes as blocks: b1b2b3 as Left Block (abbreviated as LB), b4b5b6 as Middle Block (abbreviated as MB), b7b8b9 as Right Block (abbreviated as RB), b1b4b7 as Up Block (abbreviated as UB), b2b5b8 as Central Block (abbreviated as CB) and b3b6b9 as Down Block (abbreviated as DB). A move that can be determined by scanning a single block is called a “single block move“, while a move that requires the cross reference of two blocks is called a “double block move“ sush as left block move, up block move, up-left block move, center-middle block move, down-right block move, etc.

On top of all sorts of skills, we have also developed shorthand annotations for keeping track of the order, location and type of each move! At the same time, the nomenclature of types of moves is very easy to remember and describe. For example, the method of combining column and row is called the “column row combo“, the move developed within a single box is called a “box move“ and the way of scanning among boxes and blocks is called the “scanning method“. As a rule of thumb, we shall first use the scanning method to find targets and then take a situational move among terminating, block, row, column, box and grid moves in the order of “from easy to hard“. In case of emergency, we can combine these skills to solve difficult problems. All these details will be elaborated in the following sections. Please wait and see!

However, we have to remind you at this very moment: Sudoku puzzles are so multifarious that no set of skills can crack them all. The purpose of introducing our skills is simply for you to share the wonderful experience and save your time for futile trials. The real deal will all depend on your ingenuity in using the skills, which indeed is the overwhelming charm of Sudoku!

Section 1: Basic skills practice ranges from easy to hard

For beginners

We shall now use Figure 2 to demonstrate how to fill numbers into unfilled grids of Figure 1 and finally attain Figure A.

Figure 2

	5*	7*	1*		9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*
4*		2*	3*		7*	9*	5*	1*
		9*	2*	1_1	4*	5*	8_3	7*
		1*			5*	6*	2_2	4*
5*		4*	6*		8*	3*	1*	9*
2*	4*	6*	5*		1*	8*		3*
1*	3*	8*		4*	6*	2*		5*
7*	9*	5*	8*	2*	3*	1*	4*	6*

Readers are urged to prepare Figure 1 and then follow suite. In this section we use such a simple puzzle as in Figure 1, because we would like the beginners to start up with a solid foundation.

Usage of basic scanning method, terminating and block moves:

First scan Figure 1 to find all 1 from box 1 to box 9 (briefed as "scanning 1"). We soon find out box 5 needs a 1, but there are four grids yet to be filled. How to determine the whereabout of 1?

Skill 1: SINGLE BLOCK MOVE - Here we find the 1 in question can only be entered into grid (45), so as to avoid two 1's appearing either in the same row or column of central block. This is not only taking the fact that both rows 5 and 6 have 1 into consideration, but also that the 1 in row 4 can neither be placed at grid (41) or grid (42) (which would contradict with the 1 in box 2) nor at any other grids already filled; just so we can gradually recognize the relationships of numbers among rows, columns and grids in the block, which is the skill of a block move; there are altogether six single block moves: up block (UB), central block (CB), down block (DB), left block (LB), middle block (MB) and right block (RB). The above first step was using central block move to place 1 at grid (45) , abbreviated as 1_1(45)CB, where the subscript 1 indicates that this is the "first move".

Skill 2: DOUBLE BLOCK MOVE – Since each box is located in the intersection of two perpendicular blocks, we can simultaneously use both horizontal and vertical block moves to gradual find out the fillable number. There are altogether nine double block moves: up left (UL), central left (CL), down left (DL), up middle (UM), central middle (CM), down middle (DM), up right (UR), central right (CR) and down right (DR). Here we find 2 can only be entered into box 8 (grid (48) or grid (58)), because in right block both box 7 and box 9 all have 8; had we entered 2 into grid (48), it would contradict with the 2 in row 4 of central block. Hence by double scanning of central and right blocks, we can only entered 2 into grid (58), abbreviated as 2_2(58)CR.

Skill 3: TERMINATING MOVE (t)– After each move, we should scan each related row, column and box to see if there is only one grid remained to be filled; if so, we should terminate it right away by filling in the very last number so that more easy target could reveal.

Here, we can find in box 8 only grid (48) is left for 8, thus enter 8 into grid (48) for terminating box 8, abbreviated as 8_3(48)t.

We continue with block and terminating moves as follows (Figure 3).

6_{14}	5*	7*	1*		9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*
4*		2*	3*		7*	9*	5*	1*
3_{12}	6_{13}	9*	2*	1_1	4*	5*	8_3	7*

Figure 3

		1*	9_7	3_6	5*	6*	2_2	4*
5*	2_4	4*	6*	7_5	8*	3*	1*	9*
2*	4*	6*	5*	9_9	1*	8*	7_{10}	3*
1*	3*	8*	7_8	4*	6*	2*	9_{11}	5*
7*	9*	5*	8*	2*	3*	1*	4*	6*

2_4(62)CB
7_5(65)t
3_6(55)MB
9_7(54)t
7_8(84)t
9_9(75)t
7_{10}(78)t
9_{11}(88)t
3_{12}(41)CL
6_{13}(42)t
6_{14}(11)LB

Readers should finish the remaining terminating moves to come up with Figure A.

After familiarizing these basic moves, you are able to learn more advanced skills.

For veterans

We urge readers prepare a sheet like Figure 4 filled with numbers (marked with *), then enter numbers following the demonstration (Figure 5).

Note: Since there are a lot more empty grids in Figure 4 than Figure 1, we need to use the wiser row
column combo:

Figure 4

			1*				3*	2*
	1*	3*		5*				8*
					7*			1*
6*			2*		4*	5*		
		1*		7*		6*		
		4*	6*		8*			
2*	4*		5*					
1*	3*			4*		2*		
7*	9*				3*			

Skills 4 to 7 are exactly all sorts of row and column moves from easy to hard. For facilitating the interpretation, this will also be the order of our moves.

Skill 4: **SCANNING ROW MOVE (r)** – Scanning unfilled grids of each row to gradually find among unfilled numbers a potential one to fill that won’t cause any conflict with the existing numbers in each column and each box. Note: the beginners often only focused on the conflict between column and row but are “indifferent“ with that between box and row.

Skill 5: **SCANNING COLUMN MOVE (c)** – Scanning unfilled grids of each column to gradually find among unfilled numbers a potential one to fill that won’t cause any conflict with the existing numbers in each row and each box.

Skill 6: SCANNING BOX MOVE (b) – Scanning unfilled grids of each box to gradually find among unfilled numbers a potential one to fill that won’t cause any conflict with the existing numbers in each row and each column.

Skill 7: SCANNING GRID MOVE (g) – Scanning unfilled grids of each row, column and box to find gradually find among unfilled numbers a potential one to fill that won’t cause any conflict with the existing numbers in its situated row, column and box. Unlike the above row column combo, after locating a grid we still need to preclude more than one potential unfilled number. Therefore, sometimes even the veterans would find the grid move “nowhere to set foot in“. We first introduce the traditional solving methods, and later introduce our “unique secret skills“.

Skill 8: LAW OF UNIQUE SOLUTION (u) – Suppose that a number has only two potiential grids to fit in a row, column or box in attempting row column combo. If the choice of one of them would cause multiple solutions of the puzzle, then the number in question needs to be filled into the other grid.   “Sudoku“ puzzles do not allow multiple solutions.

Figure 5

			1*				3*	2*
	1*	3*		5*	2_2		6_8	8*
		2_6			7*		5_7	1*
6*			2*		4*	5*		
		1*		7*	5_3	6*		
		4*	6*		8*			
2*	4*		5*		1_5			
1*	3*		7_4	4*		2*		
7*	9*			2_1	3*			

Block moves:

After scanning 1, we find it not easy to set foot. While scanning 2, we use down middle block move to get 2_1(95)DM, up middle block move to get 2_2(26)UM, middle block move to get 5_3(56)MB and down middle block move to get 7_4(84)DM.

Scanning column move:

Since column 6 has 1,6,9 left for its intersections with rows 1,7,8 whereas rows 1,8 have 1 already, we can only fill 1 into grid (76) of row 7, 1_5(76)c6; likewise 2 in column 3 can only be filled in grid (33) of row 3, 2_6(33)c3.

Scanning box move:

Meanwhile, box 7 has 4,5,6,7,9 left for its intersections with the first three rows and column 7; whereas row 2 and column 7 have 5 already, we can only fill 5 into grid (38) of row 3, name, we have the box move 5_7(38)b7. Siminarly, we can fill 6 into grid (28) of row 2, 6_8(28)b7.

Scanning column move:

Meanwhile, row 2 has 4,7,9 left for its intersections with columns 1,4, 7; whereas columns 1, 7 have 7 already, we can only fill 7 into grid (27) of column 7, 7_9(27)r2.

In the above, each step has a unique way to go. Although such “one pigeon one hole“ moves were very efficient in the past, but down the road we shall find them useless so as to encounter stalemates. We shall use Figure 6 to demonstrate how to use the law of unique solution for resolving the stalemate currently encountered.

Figure 6

			1*				3*	2*
	1*	3*		5*	2_{2}	7_{9}	6_{8}	8*
		2_{6}			7*		5_{7}	1*
6*			2*		4*	5*		
		1*		7*	5_{3}	6*		
		4*	6*		8*			
2*	4*		5*	(6)	1_{5}			
1*	3*	(5/6)	7_{4}	4*		2*		(6/5)
7*	9*	(6/5)		2_{1}	3*			(5/6)

Meanwhile, since column 4 has 6 at grid (64), we can enter 6 of box 6 into only two grids (grids (75),(86)). Had we entered 6 in grid (75) of box 6 (as shown in Figure 6), we would encounter the dilemma that 5, 6 in column 3 of box 3 facing 6, 5 in column 9 of box 9. In this case, two pairs of 5, 6 would be interchangeable, causing the puzzle to have two solutions contradicting the law of unique solution.

The phrase "6 in grid (75), according to the law of unique solution, would cause the dilemma of double choices between 5, 6 in column 3 of box 3 and 6, 5 in column 9 of box 9 " is abbreviated as 6(75)→u56c3b3/c9b9.

Therefore, according to the law of unique solution, 6 needs to be entered into grid (86) of box 6, 6_{10}(86)b6:6(75)→u56c3b3/c9b9 (as shown in Figure B).

Figure B

		7_{18}	1*		9_{11}	4_{13}	3*	2*
9_{14}	1*	3*	4_{15}	5*	2_{2}	7_{9}	6_{8}	8*
4_{16}		2_{6}	3_{22}		7*	9_{12}	5_{7}	1*
6*		9_{17}	2*		4*	5*		
		1*	9_{19}	7*	5_{3}	6*		
		4*	6*		8*			
2*	4*		5*	9_{20}	1_{5}			
1*	3*		7_{4}	4*	6_{10}	2*		
7*	9*		8_{21}	2_{1}	3*			

Continue with terminating, single and double block, scanning column and box moves as follows (Hereafter, terminating and block moves will only be annotated, without being explained in details):

9_{11}(16)t
9_{12}(37)b7 (Beware of 9 in box 4!)
4_{13}(17)t 9_{14}(21)UB
4_{15}(24)t 4_{16}(31)UL
9_{17}(43)LB
7_{18}(13)c3 (Beware of 7 in box 3!)

Scanning column move:

Here, column 4 has only 3,8,9 left to be filled into its intersections with rows 3,5,9; since rows 3,9 have 9 already, 9 can only be entered into grid (54) of row 5, 9_{19}(54)c4 followed by 9_{20}(75)MB , 8_{21}(94)t, 3_{22}(34)t.

This remaining puzzle as shown in Figure B is for readers to complete, and we shall discuss it in the next section.

Section 2: Struggling for survival we remain unwearied

For beginners **Figure 1**

			1*		9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*
4*		2*	3*		7*	9*	5*	1*
6*		9*	2*		4*	5*		
		1*	9*	7*	5*	6*		
		4*	6*		8*			
2*	4*		5*	9*	1*			
1*	3*		7*	4*	6*	2*		
7*	9*		8*	2*	3*			

First use the method of scanning blocks, to our surprise what we learned in Chapter 1seems to fail altogether! The problem is it looks like in all rows, columns and boxes, each and every number has more than one place to set foot in, but it is not allowed to pick one at will. For example, we can not arbitrarily enter 2 into either grid (52) or (62) of column 2: neither can we enter 3 into grid (51) or (61) ; nor enter 7 into grid (42) nor (62).

We have scanned back and forth in Figure 1and suddenly have a feeling of "tricks in use always fall short"!

Don't worry! This is because of the unfamiliarity of the basic skills. With more careful scanning, we should be able to figure something out. Let us have a peace of mind and continue our drill!

Scanning row move:

At this time, column 7 has 1,3,8 left for its intersections with rows 6,7, 9; whereas rows 6, 9 have 8 already, 8 can only be filled into grid (77) of row 7, 8_1(77)c7, followed by 8_2(83)DB.

Scanning box move:

Meanwhile, box 3 has 5,6 left for its intersections with rows 7,9; whereas row 7 has 5 already, 5 can only be filled into grid (93) of row 9, 5_3(93)b3, followed by 6_4(73)t.

Figure 2

Block and terminating moves:

7_5(13)t 6_6(99)DR
4_7(98)DR 1_8(97)t
3_9(67)t 3_{10}(79)DR
7_{11}(78)t 4_{12}(59)CR
5_{13}(89)DR 9_{14}(88)t
9_{15}(69)CR 7_{16}(49)t

		7_5	1*		9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*
4*		2*	3*		7*	9*	5*	1*
6*		9*	2*		4*	5*		7_{16}
		1*	9*	7*	5*	6*		4_{12}
		4*	6*		8*	3_9		9_{15}
2*	4*	6_4	5*	9*	1*	8_1	7_{11}	3_{10}
1*	3*	8_2	7*	4*	6*	2*	9_{14}	5_{13}
7*	9*	5_3	8*	2*	3*	1_8	4_7	6_6

The steps that follow will be completed in Figure 3.

8_{26}	5_{28}	7_5	1*	6_{30}	9*	4*	3*	2*
9*	1*	3*	4*	5*	2*	7*	6*	8*

4*	6_{29}	2*	3*	8_{31}	7*	9*	5*	1*
6*	8_{24}	9*	2*	3_{20}	4*	5*	1_{23}	7_{16}
3_{25}	2_{18}	1*	9*	7*	5*	6*	8_{27}	4_{12}
5_{22}	7_{17}	4*	6*	1_{21}	8*	3_{9}	2_{19}	9_{15}
2*	4*	6_{4}	5*	9*	1*	8_{1}	7_{11}	3_{10}
1*	3*	8_{2}	7*	4*	6*	2*	9_{14}	5_{13}
7*	9*	5_{3}	8*	2*	3*	1_{8}	4_{7}	6_{6}

Figure 3

Block and terminating moves:

7_{17}(62)CL 2_{18}(52)CL
2_{19}(68)CR 3_{20}(45)CM
1_{21}(65)t 5_{22}(61)t
1_{23}(48)CR 8_{24}(42)t
3_{25}(51)t 8_{26}(11)t
8_{27}(58)t 5_{28}(12)UL
6_{29}(32)t 6_{30}(15)t
8_{31}(35)t

Congratuations! You have finally familiarized a string of moves to finish this puzzle! We shall provide you the following brand new puzzle (Figure 4) to test you ability!

For veterans

Figure 4

4*		2*			1*	3*		7*
			9*	2*			5*	
				8*			2*	
6*		7*				1*		2*
3*		1*				4*		9*
				4*				
	1*	4*		5*	6*			
8*		5*	3*			2*	4*	6*

Since each box has too many empty grids, we realize that the familiar block, scanning row , column and box moves are indeed no longer useful! After all is there any trick that is left off? Ah! Just occurred to us! Ought to follow the order of "from easy to hard", it must be time for scanning grid moves to take the stage!

Scanning grid moves can be used among various combinations of row, column and box, which is to utilize the numbers already filled (almost all numbers from 1 to 9 are complete) in the related row, column and box where a certain grid is situated to filter out the one and only fillable number.

4*		2*	5_{2}	6_{1}	1*	3*		7*
			9*	2*			5*	
				8*			2*	
6*		7*				1*		2*
						5?		***5?***
3*		1*		7_{3}		4*		9*
				4*				

Figure 5

	1*	4*		5*	6*			
8*		5*	3*			2*	4*	6*

Scanning grid moves:

Since row 1 where grid (15) is situated has 1,2,3,4,7, column 5 has 5,8 and box 4 has 9, only 6 is left to be filled into grid (15). 6_1(15)g; since row 1 where grid (14) is situated has 1,2,3,4,6,7, column 4 has 9 and box 4 has 8, only 5 is left to be filled into grid (14). 5_2(14)g; since row 6 where grid (65) is situated has 1,3,9 and column 5 has 2,4,5,6,8, only 7 is left to be filled into grid (65). 7_3(65)g.

Skill 9: MAGIC MIRROR FOR REVEALING NUMBERS – When using row column combo, we need to scan filled numbers in each row and each column; but some of them are "invisible" (namely, not yet filled), which need to be revealed by a magic mirror. For example, row 5 of box 8 does have an invisible 5; this is because there is already a 5 in column 8 outside of box 8, hence the 5 of box 8 is forced to squize into (not in column 8) grid (57) or (59) (as shown in Figure 5).

Scanning column move + magic mirror:

At this time, column 1 has 1,2,5,7,9 left for its intersections with rows 2,3,5,7, 9; whereas row 2, box 3 have 5 already, and row 5 has an invisible 5 as well (since there is already a 5 in column 8 outside of box 8 as shown in Figure 5), hence 5 can only be filled into grid (31) of row 3, 5_4(31)c1: 5(28)→5r5b8. (as shown in Figure 6)

Block moves: **Figure 6**

1_5(21)LB 1_6(39)UR 1_7(78)DR
1_8(95)DM 1_9(54)CM
4_{10}(29)RB 6_{11}(64)CM

Scanning row move + magic mirror:

At this time, row 9 has 7,9 left for its intersections with columns 2,6; whereas column 2 of box 1 has an invisible 7 (since there is already a 7 in column 3 outside of box 1), hence 7 can only be filled into grid (96) of column 6,

4*		2*	5_2	6_1	1*	3*		7*
1_5			9*	2*			5*	4_{10}
5_4				8*			2*	1_6
6*		7*				1*		2*
			1_9					
3*		1*	6_{11}	7_3		4*		9*
				4*			1_7	
	1*	4*		5*	6*			
8*		5*	3*	1_8		2*	4*	6*

4*		2*	5_2	6_1	1*	3*		7*
1_5	7_{19}		9*	2*	3_{16}		5*	4_{10}
5_4			7_{14}	8*	4_{15}		2*	1_6
6*		7*	4_{17}			1*		2*
	4_{18}		1_9					
3*		1*	6_{11}	7_3		4*		9*
				4*			1_7	

7_{12}(96)r9: 7(43)→7c2b1.

Figure B

	1*	4*		5*	6*			
8*	9_{13}	5*	3*	1_8	7_{12}	2*	4*	6*

Block and terminating moves:

9_{13}(92)t 7_{14}(34)MB
4_{15}(36)UB 3_{16}(26)t
4_{17}(44)MB 4_{18}(52)CL
7_{19}(22)UL

The remaining puzzle as shown in Figure B is left for reader to complete. We shall discuss it in the next section.

Section 3: Back and forth drills make skills turn shrewd

For beginners **Figure 1**

With block scanning failed, try scanning row or column move. The juncture of row 1 and column 2 is rightfully a good example of unfilled numbers not too overlapping: row 1 has only two grids (12),(18) left for 8,9, whereas column 2 has five grids (12),(32),(42),(62),(72) left for 2,3,5,6,8; since both have only 8 overlaped.We thus can start with the scanning row move (see Figure 2).

4*		2*	5*	6*	1*	3*		7*
1*	7*		9*	2*	3*		5*	4*
5*			7*	8*	4*		2*	1*
6*		7*	4*			1*		2*
	4*		1*					
3*		1*	6*	7*		4*		9*
				4*			1*	
	1*	4*		5*	6*			
8*	9*	5*	3*	1*	7*	2*	4*	6*

Figure 2

4*	8_1	2*	5*	6*	1*	3*	9_2	7*
1*	7*	6_4	9*	2*	3*	8_3	5*	4*
5*	3_{12}	9_{11}	7*	8*	4*	6_5	2*	1*
6*		7*	4*			1*		2*
	4*	8_7	1*			7_9	6_8	
3*		1*	6*	7*		4*		9*
	6_6	3_{13}		4*			1*	
	1*	4*		5*	6*		7_{10}	
8*	9*	5*	3*	1*	7*	2*	4*	6*

Scanning row move:

Since row 1 has 8,9 left for filling in its intersections with columns 2,8 whereas columns 2 has 9 already, only 8 can be filled into grid (12) of columns 2, 8_1(12)g, followed by 9_2(18)t.

Block and terminating moves;

8_3(27)UB 6_4(23)t 6_5(37)t
6_6(72)LB 8_7(53)LB
6_8(58)CB
7_9(57)CR 7_{10}(88)RB

Scanning row move:

Since row 3 has 3,9 left for filling in its intersections with columns 2,3 whereas columns 2 has 9 already, only 9 can be filled into grid (33) of row 3, 9_{11}(33)r3 , 3_{12}(32)t, 3_{13}(73)t.

For the rest we urge readers to complete.

3_{14}(89)DB 3_{15}(48)CR 8_{16}(68)t 5_{17}(59)t 8_{18}(79)t 3_{19}(55)CM 9_{20}(45)t
5_{21}(77)DR 9_{22}(87)t 7_{23}(71)DL 2_{24}(81)t 9_{25}(51)t 2_{26}(56)t 8_{27}(84)t etc.

For veterans

Figure 3

5*							9*	8*
1*		8*		3*				
	4*				6*		3*	1*
2*	6*	4*	3*		5*		1*	
	5*	1*				3*		2*
			2*				5*	4*
7*			9*				2*	
				7*		1*		
4*	1*							3*

Upon failing with the scanning block method, how shall we set foot in? Please look at Figure 4.

Scanning column move:

Since column 1 has 3,6,8,9 left for filling in its intersections with rows 3,5,6,8 whereas row 3 has 3, 6 already and box 1 has 8, 9 can only be filled into grid (31) which is situated simultaneously in row 3 and box 1, 9_1(31)g followed by 9_2(26)UM.

Scanning row move:

Since row 5 has 4,6,7,8,9 left for filling in its intersections with columns 1,4,5,6,8 whereas columns 1,4,6,8 have 9 already, 9 can only be filled into grid (55) of row 5, 9_3(55)r5.

Figure 4

5*	3_5	6_4					9*	8*
1*		8*		3*	9_2			
9_1	4*				6*		3*	1*
2*	6*	4*	3*		5*		1*	
	5*	1*		9_3		3*		2*
			2*				5*	4*
7*			9*				2*	
6_6				7*		1*		
4*	1*							3*

Scanning box move:

Since box 1 has 2,3,6 left for filling in its intersections with column 2, rows 1,3 whereas column

2, row 3 have 6 already, 6 can only be filled into grid (13) which is situated in row 1 but not column 2, 6_4(13)b1, followed by 3_5(12)UB, 6_6(81)LB.

Figure 5

5*	3_5	6_4	1_{16}	4_{18}			9*	8*
1*		8*		3*	9_2			
9_1	4*		8_{10}		6*		3*	1*
2*	6*	4*	3*	8_9	5*		1*	
8_8	5*	1*		9_3		3*	6_{12}	2*
3_7			2*	6_{13}	1_{14}	8_{11}	5*	4*
7*			9*	1_{15}			2*	
6_6				7*		1*		
4*	1*		6_{17}					3*

Scanning column move:
Since column 1 has 3,8 left for filling in its intersections with rows 5,6, whereas row 5 has 3 already, 3 can only be filled into grid (61) of row 6, 3_7(61)c1, followed by 8_8(51)t.

Scanning grid move:
At this time, row 4 where grid (45) is situated has 1, 2, 3, 4, 5, 6 and column 5 has 7, 9, only 8 is left for filling into grid (45), 8_9(45)g.

Block moves:

8_{10}(34)UM 8_{11}(67)CB 6_{12}(58)b8 6_{13}(65)CM 1_{14}(66)CB 1_{15}(75)DM 1_{16}(14)UM 6_{17}(94)DM

Scanning column move: Since column 5 has 2,4,5 left for filling in its intersections with rows 1,3,9 whereas rows 3,9 have 4 already, 4 can only be filled into grid (15) of row 5, 4_{18}(15)c5.

We shall resolve the next touchy move by using proof by contradiction.

Skill 10: PROOF BY CONTRADICTION–During the hardship of “tying our hands up“ for utilizing our skills, if we find two possible moves, it does not necessarily mean the puzzle has two solutions. Hence we need to try both ways out. If one of them would lead to a “dead end“ while the other leads to a solution, then we can prove that there is no contradiction with law of unique solution. Here is an example (see Figure 6 and Figure B).

Figure 6

5*	3_5	6_4	1_{16}	4_{18}	(7_3)		9*	8*
1*	(7_1)	8*		3*	9_2			
9_1	4*	(2_2)	8_{10}		6*		3*	1*
2*	6*	4*	3*	8_9	5*		1*	
8_8	5*	1*		9_3		3*	6_{12}	2*
3_7			2*	6_{13}	1_{14}	8_{11}	5*	4*
7*			9*	1_{15}			2*	
6_6				7*		1*		
4*	1*		6_{17}					3*

Scanning box move + proof by contradiction:

At this time, box 1 has only grids (22) and (33) left for 2, 7. Had we entered 7 into (22), then we would use up-middle block move to enter 7 of box 4 into grid (16) and 2 into grid (24) or (35); as a result, 2 would conflict the 2 in row 3 or column 4, abbreviated as 7(22)→2(33)→7(16)→no2b4.

Therefore, to avoid the conflict of 2 we should enter 2 into grid (22) of box 1, abbreviated as 2_{19}(22)b1:7(22)→2(33)→7(16)→no2b4. Then enter 7 into grid (33), 7_{20}(33)t (as shown in Figure B).

Figure B

Block or terminating moves:

7_{21}(62)LB 9_{22}(63)t
9_{23}(82)DL 8_{24}(72)t
9_{25}(97)DR 9_{26}(49)RB
7_{27}(47)t

The remaining puzzle as shown in Figure B should be completed by readers. We shall discuss it in the next section.

5*	3_{5}	6_{4}	1_{16}	4_{18}			9*	8*
1*	2_{19}	8*		3*	9_{2}			
9_{1}	4*	7_{20}	8_{10}		6*		3*	1*
2*	6*	4*	3*	8_{9}	5*	7_{27}	1*	9_{26}
8_{8}	5*	1*		9_{3}		3*	6_{12}	2*
3_{7}	7_{21}	9_{22}	2*	6_{13}	1_{14}	8_{11}	5*	4*
7*	8_{24}		9*	1_{15}			2*	
6_{6}	9_{23}			7*		1*		
4*	1*		6_{17}			9_{25}		3*

Section 4 : Various complicated problems can all be settled

For beginners

In the previous section, we have encountered quite a few hardships and utilized all the learned skills to the maximum extreme. Keeping cool and calm, training efficient solving skills and maintaining good solving habbits are the necessary ways to learn “Sudoku“!

Now, this remaining not too hard puzzle might as well be a good opportunity to test your “Sudoku“ ability.

5*	3*	6*	1*	4*			9*	8*
1*	2*	8*		3*	9*			
9*	4*	7*	8*		6*		3*	1*
2*	6*	4*	3*	8*	5*	7*	1*	9*
8*	5*	1*		9*		3*	6*	2*
3*	7*	9*	2*	6*	1*	8*	5*	4*
7*	8*		9*	1*			2*	
6*	9*			7*		1*		

Figure 1

4*	1*		6*			9*		3*

In row 1, we have left with (16), (17) for 2, 7. Since there is 7 in column 7 outside of row 1, we have 7_1(16)r1, followed by 2_2(17)t.

The rest are block and terminating moves.

2_3(35)UB 5_4(24)t 5_5(95)t 5_6(37)t 7_7(54)MB 4_8(56)t 4_9(84)t 4_{10}(77)DR
6_{11}(27)t 4_{12}(28)RB 7_{13}(29)t 6_{14}(79)DR 5_{15}(89)t 5_{16}(73)DB 3_{17}(76)t

Figure 2

5*	3*	6*	1*	4*	7_1	2_2	9*	8*
1*	2*	8*	5_4	3*	9*	6_{11}	4_{12}	7_{13}
9*	4*	7*	8*	2_3	6*	5_6	3*	1*
2*	6*	4*	3*	8*	5*	7*	1*	9*
8*	5*	1*	7_7	9*	4_8	3*	6*	2*
3*	7*	9*	2*	6*	1*	8*	5*	4*
7*	8*		9*	1*		4_{10}	2*	6_{14}
6*	9*		4_9	7*		1*		
4*	1*		6*	5_5		9*		3*

For veterans

Figure 3

	3*			9*			8*	
9*								4*
		1*	5*	(4_4)	2*	6*		
		2*		1*		5*		
3*				(2_3)				1*
		4*		6*		3*		
		9*	6*		4*	2*		
5*	(2_1)							7*
(4_2)	8*			7*			1*	

We immediately encounter a stalemate, for which we need to utilize “magic mirror“ and “proof by contradiction“ simultaneously!

Scanning box move + magic mirror+ proof by contradiction:
Though box 3 has six unfilled grids, but since both row 7 and column 3 have 2,4 already, we use the magic mirroir to reflect the hidden 2,4 at grid (82) or (91).

At this time if we were to enter 2_1 into grid (82), then 4_2 would have to be entered into grid (91); according to the scanning column move, 2_3 and 4_4 of column 5 could only be entered into (55) and (35) consecutively.
As a result, the 4 of box 1 according to up-left block move, could neither be in rows 2 and 3 nor be in columns 1 and 3, becoming no place to go, 2(82)b3→4(91)→2(55)→4(35)→no4r1. Hence, by proof of contradiction, we should enter 2 into grid (91),
2_1(91)b3:2(82)b3→4(91)→2(55)→4(35)→no4r1.

Figure 4

At this juncture, the way to go about it is not unique. We shall discuss the remaining puzzle with the move 2_1(91)b3 in the next section. Meanwhile, we shall first look at Figure 4 and then Figure 5 to demonstrate another kind of skill: “spiderman’s kungfu“!

2/6	3*		4_1	9*			8*	
9*	6/2							4*
4?	4?	1*	5*		2*	6*		
		2*		1*		5*		
3*								1*
		4*		6*		3*		
		9*	6*		4*	2*		
5*	4/2	6?						7*
2/4	8*	6?		7*			1*	

Scanning box move + magic mirror + block move:
As illustrated above, box 6 still has six unfilled grids. But since column 3 and row 7 both have 2,4 already, we can use the magic mirroir to reflect the hidden 2, 4 at grid (82) or (91); in addition, since row 7 has 6 already, reflecting that column 3 of box 3 has hidden 6 as well. At this time, since column 3 and row 3 both have 2,6, reflecting that grids (11) and (22) of box 1 must have hidden 2, 6; as a result, box 1 has only row 3 and column 3 unfilled grids left to fill in numbers; but since column 3 outside of the box has 4 already, row 3 of box 1 must have a hidden 4. Now according to up-middle block move, since rows 2,3 and column 7 all have 4, so 4 can only be entered into grid (14), 4_1(14)UM (as shown in Figure 4).

We only provide annotations for following moves.

4_2(55)MB 4_3(48)CR 5_4(75)c5 2_5(85)c5 2_6(91)DL 2_7(22)UL 2_8(19)UR
4_9(82)b3 4_{10}(31)UL 4_{11}(97)DR 5_{12}(28)b7 5_{13}(13)UL 5_{14}(99)DR

	3*	5_{13}	4_1	9*			8*	2_8
9*	2_7						5_{12}	4*
4_{10}		1*	5*		2*	6*		

Figure 5

We continue the following moves as shown in Figure B.

6_{15}(88)b9	6_{16}(93)DB
6_{17}(49)CR	6_{18}(52)CL
6_{19}(11)LB	6_{20}(26)UM
3_{21}(83)LB	5_{22}(62)CL
1_{23}(61)CB	1_{24}(72)LB
7_{25}(71)t	8_{26}(41)t
5_{27}(56)CM	8_{28}(23)LB
7_{29}(32)t	9_{30}(42)t
7_{31}(53)t	8_{32}(35)UB
3_{33}(25)t	

		2*		1*		5*	4_{3}	
3*				4_{2}				1*
		4*		6*		3*		
		9*	6*	5_{4}	4*	2*		
5*	4_{9}			2_{5}				7*
2_{6}	8*			7*		4_{11}	1*	5_{14}

Figure B

This remaining puzzle of Figure B is left for readers to complete.

Congratulations! You have already turned from beginners to veterans! From now on, you'll formally set foot in "Sudoku jungle"!

Yet, the genuine challenges are about to start!

6_{19}	3*	5_{13}	4_{1}	9*			8*	2_{8}
9*	2_{7}	8_{28}		3_{33}	6_{20}		5_{12}	4*
4_{10}	7_{29}	1*	5*	8_{32}	2*	6*		
8_{26}	9_{30}	2*		1*		5*	4_{3}	6_{17}
3*	6_{18}	7_{31}		4_{2}	5_{27}			1*
1_{23}	5_{22}	4*		6*		3*		
7_{25}	1_{24}	9*	6*	5_{4}	4*	2*		
5*	4_{9}	3_{21}		2_{5}			6_{15}	7*
2_{6}	8*	6_{16}		7*		4_{11}	1*	5_{14}

Sudoku-Part 2 will be released on June 15, 2017.